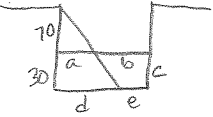


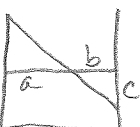
Related Rates Test Review Answer Key

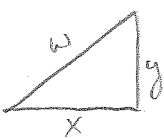
① $y = 50$ $\frac{dx}{dt} = -5$, $x = 30$ a) $\frac{dy}{dt} = ?$ $x^2 + y^2 = 50^2$ $\frac{dy}{dt} = \frac{x \frac{dx}{dt}}{y} = \frac{(30)(-5)}{40}$
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$
 $y = \frac{-15 \text{ ft}}{4 \text{ sec}}$

b) $A = \frac{1}{2}xy$
 $\frac{dA}{dt} = \frac{1}{2} \frac{dx}{dt} y + \frac{1}{2} x \frac{dy}{dt} = \frac{1}{2}(-5)(40) + \frac{1}{2}(30)\left(\frac{-15}{4}\right) = \frac{-625}{4} \frac{\text{ft}^2}{\text{sec}}$

②  $\frac{da}{dt} = 2$ a) $\frac{dd}{dt} = ?$ $\frac{a}{70} = \frac{d}{100} \rightarrow 100a = 70d$ $\frac{dd}{dt} = \frac{100(2)}{70} = \frac{20 \text{ ft}}{7 \text{ sec}}$
 $100 \frac{da}{dt} = 70 \frac{dd}{dt}$

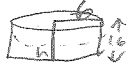
b) $\frac{a}{70} = \frac{50}{100}$ $a = 35 \text{ ft}$

c)  $b = 10$ $a = 40$ $\frac{a}{70} = \frac{b}{c}$ $ac = 3500 - 70a$ $\frac{da}{dt} \cdot c + a \frac{dc}{dt} = -70 \frac{da}{dt}$
 $\frac{40}{70} = \frac{10}{c} \rightarrow c = \frac{35}{2}$
 $(2)\left(\frac{35}{2}\right) + 40 \frac{dc}{dt} = (-70)(2)$
 $\frac{dc}{dt} = \frac{-140 - 35}{40} = \frac{-35}{8} \frac{\text{ft}}{\text{sec}}$

③  $\frac{dx}{dt} = 6$ $y = 30$ $x = 40$ $w = 50$ $\frac{dw}{dt} = ?$ moving at a rate of $\frac{35 \text{ ft}}{8 \text{ sec}}$ up the wall
 $\frac{x}{w} = \frac{40}{50}$ $50x = 40w$ $50 \frac{dx}{dt} = 40 \frac{dw}{dt}$ $\frac{dw}{dt} = \frac{50(6)}{40} = \frac{15 \text{ ft}}{2 \text{ sec}}$

④ $\frac{dd}{dt} = -.02$ a) $A = \pi \left(\frac{1}{2}d\right)^2 = \frac{1}{4}\pi d^2 \rightarrow \frac{dA}{dt} = \frac{1}{2}\pi d \frac{dd}{dt} = \frac{1}{2}\pi(100)(-.02) = -\pi \frac{\text{cm}^2}{\text{sec}}$

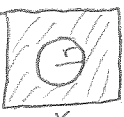
b) $C = \pi d$ $\frac{dC}{dt} = \pi \frac{dd}{dt} = -.02\pi \frac{\text{cm}}{\text{sec}}$

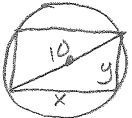
⑤ $\frac{dV}{dt} = -5$, $r = 6$, $h = 3$, $\frac{dh}{dt} = ?$  $V = \pi r^2 h = 36\pi h$ $\frac{dV}{dt} = 36\pi \frac{dh}{dt} \rightarrow -5 = 36\pi \frac{dh}{dt}$
 $\frac{dh}{dt} = \frac{-5}{36\pi} \frac{\text{cm}}{\text{hr}}$

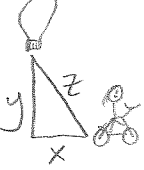
⑥ $\frac{dx}{dt} = -2$, $\frac{dy}{dt} = 2$ a) $A = xy \rightarrow \frac{dA}{dt} = \frac{dx}{dt} y + x \frac{dy}{dt} = -2(5) + 2(12) = 14 \frac{\text{cm}^2}{\text{sec}}$
 $x = 12$, $y = 5$

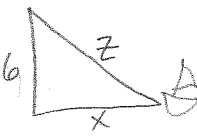
b) $P = 2x + 2y \rightarrow \frac{dP}{dt} = 2 \frac{dx}{dt} + 2 \frac{dy}{dt} = 2(-2) + 2(2) = 0 \frac{\text{cm}}{\text{sec}}$

c) $x^2 + y^2 = z^2 \rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt} \rightarrow \frac{dz}{dt} = \frac{2(12)(-2) + 2(5)(2)}{2(13)}$

⑦  $A = x^2 - \pi r^2$ $\frac{dA}{dt} = 2x \frac{dx}{dt} - 2\pi r \frac{dr}{dt} = 2(12)(2) - 2\pi(3)(4) = 48 - 24\pi \frac{\text{in}^2}{\text{min}}$
 $r = 3$ $\frac{dr}{dt} = 4$
 $x = 12$ $\frac{dx}{dt} = 2$

8)  $\frac{dx}{dt} = -2$ $A = xy = x\sqrt{100-x^2}$ $\frac{dA}{dt} = \frac{dx}{dt}\sqrt{100-x^2} + x\left(\frac{1}{2}\right)(100-x^2)^{-\frac{1}{2}}(-2x\frac{dx}{dt})$
 $x=6$ $x^2+y^2=100$ $y=8$ $y=\sqrt{100-x^2}$
 $\rightarrow = (-2)\sqrt{100-36} + 6\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{100-36}}\right)(-2)(6)(-2)$
 $= \boxed{-7 \frac{\text{in}^2}{\text{sec}}}$

9)  $\frac{dy}{dt} = 1$, $\frac{dx}{dt} = 17$, $\frac{dz}{dt} = ?$, $y = 65 + 3 = 68$, $x = 51$, $z = 85$
 $x^2 + y^2 = z^2$
 $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$
 $\rightarrow \frac{dz}{dt} = \frac{51(17) + 68(1)}{85} = \boxed{11 \frac{\text{ft}}{\text{sec}}}$

10)  $\frac{dz}{dt} = -2$ a) $\frac{dx}{dt} = ?$ $x^2 + 6^2 = z^2$
 $z = 10$ $x = 8$ $2x\frac{dx}{dt} = 2z\frac{dz}{dt}$
 $\rightarrow \frac{dx}{dt} = \frac{10(-2)}{8} = \boxed{-\frac{5}{2} \frac{\text{ft}}{\text{sec}}}$
 b) $A = \frac{1}{2}(6)(x) = 3x$ $\frac{dA}{dt} = 3\frac{dx}{dt} = 3\left(-\frac{5}{2}\right) = \boxed{-\frac{15}{2} \frac{\text{ft}^2}{\text{sec}}}$

11) $f(-5) = 10$ point: $(-5, 10)$ $f'(-5) = \frac{-35}{20} = -\frac{7}{4}$
 $f(x) = \sqrt{3x^2 - 5x}$
 $f'(x) = \frac{1}{2}(3x^2 - 5x)^{-\frac{1}{2}}(6x - 5) = \frac{6x - 5}{2\sqrt{3x^2 - 5x}}$
 $f(-4.9) \approx \frac{407}{40}$
 $y - 10 = -\frac{7}{4}(x + 5)$
 $y - 10 = -\frac{7}{4}(-4.9 + 5)$
 $y = \frac{407}{40}$

12) $f'(1) = 5$ a) $y - 2 = 5(x - 1)$ $f(0.9) \approx 5(0.9) - 3 = 1.5$
 $f(1) = 2$ $y = 5x - 3$ $f(1.1) \approx 5(1.1) - 3 = 2.5$
 b) estimates are too small because $f(x)$ is concave up

13) $\lim_{x \rightarrow 3} \frac{4x^2 - 5x}{1 - 3x^2} = \boxed{\frac{21}{-26}}$ $\lim_{x \rightarrow 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$
 $= \lim_{x \rightarrow 1} \frac{20x^3 - 8x}{-1 - 27x^2} = \frac{12}{-28} = \boxed{-\frac{3}{7}}$