Chapter 5 Integral Review

1. Evaluate the integral: $\int_1^{5e} \frac{1}{x}\,dx$.

(a) $\frac{1}{5e} - 1$ (b) 0 (c) ∞

(d) $1 + \ln 5$ (e) None of these

2. Evaluate the integral: $\int \frac{x+2}{x+1}\,dx$.

(a) $\frac{x^2+4x}{x^2+2x} + C$ (b) $2x + C$ (c) $x + C$

(d) $x + \ln|x+1| + C$ (e) None of these

15. Evaluate the integral: $\int x \cot x^2\,dx$.

(a) $\frac{1}{2}x^2 \sec^2 x^2 + C$ (b) $\frac{1}{4}x^2 \ln|\sin x^2| + C$ (c) $x \cot x^2 \csc x^2 + C$

(d) $\frac{1}{2}\ln|\sin x^2| + C$ (e) None of these

16. Evaluate the integral: $\int \frac{\cos^3 x - \sin^2 x}{\cos^2 x}\,dx$.

(a) $\frac{\cos^2 x}{2} - \tan x + x + C$ (b) $\sin x - \sec x + C$ (c) $\sin x - \tan x + x + C$

(d) $\sin x - \frac{\tan^3 x}{3} + C$ (e) None of these

17. Evaluate the integral: $\int \sec 2x\,dx$.

19. Evaluate the integral: $\int \frac{\ln\sqrt{x}}{x}\,dx$.

21. Evaluate the integral: $\int \frac{1-\sin\theta}{\cos\theta}\,d\theta$.

23. Solve the differential equation: $\frac{dy}{dx} = \frac{3x}{1-x^2}$.

1. Evaluate: $\int \frac{x+3}{x^2+9}\,dx$.

(a) $\ln|x-3| + C$ (b) $\frac{1}{3}\arctan\frac{x}{3} + C$ (c) $\frac{1}{2}\ln(x^2+9) + \arctan\frac{x}{3} + C$

(d) $\ln(x^2+9) + \frac{1}{3}\arctan\frac{x}{3} + C$ (e) None of these

2. Evaluate: $\int \frac{dx}{\sqrt{8 + 2x - x^2}}$

(a) $\ln\sqrt{8 + 2x - x^2}$

(b) $\arcsin\frac{x-1}{3} + C$

(c) $\sqrt{8 + 2x - x^2} + C$

(d) $\frac{1}{3}\operatorname{arcsec}\frac{x-1}{3} + C$

(e) None of these

3. Evaluate: $\int \frac{x + 2}{\sqrt{4 - x^2}}\, dx.$

(a) $-\frac{1}{2}\sqrt{4 - x^2} + 2\arcsin\frac{x}{2} + C$

(b) $-\sqrt{4 - x^2} + 2\arcsin\frac{x}{2} + C$

(c) $\ln|2 - x| + C$

(d) $x^2 + 2x + \arcsin\frac{x}{2} + C$

(e) None of these

4. Evaluate: $\int \frac{5}{x^2 + 6x + 13}\, dx.$

(a) $5\ln|x^2 + 6x + 13| + C$

(b) $5\left(\frac{x^3}{3} + 3x^2 + 13x\right) + C$

(c) $\frac{5}{2}\arctan\frac{x+3}{2} + C$

(d) $-\frac{5}{x} + \frac{5}{6}\ln|x| + \frac{5}{13}x + C$

(e) None of these

5. Find the indefinite integral: $\int \frac{x}{16 + x^4}\, dx.$

(a) $\frac{1}{2}\arcsin\frac{x^2}{4} + C$

(b) $\frac{1}{8}\arctan\frac{x^2}{4} + C$

(c) $\frac{1}{4}\arctan\frac{x^2}{4} + C$

(d) $\frac{1}{8}\operatorname{arcsec}\frac{x^2}{4} + C$

(e) None of these

7. **Evaluate the definite integral:** $\int_1^{\sqrt{e}} \frac{-4x}{x^2}\, dx.$

(a) -1

(b) -6

(c) -4

(d) -2

(e) None of these

10. Evaluate the integral: $\int \frac{1}{2x\sqrt{4x^2 - 1}}\, dx.$

(a) $\operatorname{arcsec}|2x| + C$

(b) $\frac{1}{2}\arcsin|2x| + C$

(c) $\frac{1}{8}\sqrt{4x^2 - 1} + C$

(d) $\frac{1}{2}\operatorname{arcsec}|2x| + C$

(e) None of these

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10. Evaluate the integral: $\int \frac{3x^2 + 3x + 3}{x^2 + 1}\,dx$.

(a) $3x + 3\ln(x^2 + 1) + C$ (b) $3 + \frac{3}{2}\ln(x^2 + 1) + C$ (c) $3x + \frac{3}{2}\ln(x^2 + 1) + C$

(d) $3 + 3\ln(x^2 + 1) + C$ (e) None of these

17. Evaluate the integral: $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

(a) $2e^{\sqrt{x}} + C$ (b) $\frac{1}{2}e^{\sqrt{x}} + C$ (c) $\sqrt{x}\,e^{\sqrt{x}} + C$

(d) $\sqrt{x}\,e^{\sqrt{x}+1} + C$ (e) None of these

21. Evaluate the indefinite integral: $\int \frac{1}{x^2 e^{5/x}}\,dx$.

(a) $\frac{1}{5}e^{5/x} + C$ (b) $\frac{1}{5}xe^{5/x} + C$ (c) $\frac{1}{5}e^{-5/x} + C$

(d) $\frac{1}{5}xe^{-5/x} + C$ (e) None of these

23. Calculate the area of the region bounded by $y = e^{2x}$, $y = 0$, $x = 1$, $x = 4$.

24. Differentiate: $f(x) = \ln(e^{-x^2})$.

(a) e^{x^2} (b) $-2xe^{2x^2}$ (c) $-2x$

(d) $-2xe^{-x^2}$ (e) None of these

17. Evaluate the integral: $\int x3^{x^2}\,dx$.

(a) $\left(\frac{x^2}{2}\right)3^{(x^3/3)} + C$ (b) $\left(\frac{\ln 3}{2}\right)3^{x^2} + C$ (c) $\frac{3^{x^2}}{2\ln 3} + C$

(d) $\frac{1}{2}(3^{x^2}) + C$ (e) None of these

12. A radioactive element has a half-life of 50 days. What percentage of the original sample is left after 60 days?

(a) 43.53% (b) 49.56% (c) 37.50%

(d) 25.00% (e) None of these

12. Evaluate the definite integral: $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{3}{\sqrt{4 - 9x^2}}\,dx$.

25. Evaluate the indefinite integral: $\int \frac{1}{(x - 3)\sqrt{x^2 - 6x + 5}}\,dx$.

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2. A deposit of $1000 is made into a fund with an annual interest rate of 5 percent. Find the time (in years) necessary for the investment to triple if the interest is compounded continuously. Round your answer to 2 decimal places.

(a) 30.00 years (b) 15.00 years (c) 22.24 years

(d) 21.97 years (e) None of these

6. Use integration to find a general solution to the differential equation $y' = \dfrac{2}{\sqrt{1-x^2}} + x$.

(a) $\arcsin\left(\dfrac{x}{2}\right) + C$ (b) $2 \arcsin x + \dfrac{x^2}{2} + C$ (c) $2 \arcsin x + x^2 + C$

(d) $2 \arcsin x^2 + \dfrac{x^2}{2} + C$ (e) None of these

7. Use integration to find a general solution to the differential equation $y' = x\sqrt{x+1}$.

(a) $\frac{2}{5}(x+1)^{3/2}\left(x - \frac{2}{3}\right) + C$ (b) $\frac{2}{5}(x+1)^{5/2} - \frac{1}{3}(x+1)^{3/2} + C$

(c) $(x+1)^{5/2} + (x+1)^{3/2} + C$ (d) $2(x+1)^{3/2}(3x-2) + C$

(e) None of these

19. Solve the differential equation: $\sqrt{x^2 - 1}\, y' = \dfrac{y}{x}$.

21. The rate of change of y with respect to x is inversely proportional to the square root of y.

a. Write a differential equation for the given statement.

b. Solve the differential equation in part a.

24. The number of fruit flies increases according to the law of exponential growth. If initially there are 10 fruit flies and after 6 hours there are 24, find the number of fruit flies after t hours.

(a) $y = 10e^{\ln(12/5)t/6}$ (b) $y = 10e^{\ln(12/5)t}$ (c) $y = 10e^{-\ln(12/5)t/6}$

(d) $y = 10e^{(\ln 12)t/6}$ (e) None of these

35. Find the general solution of the differential equation: $\dfrac{y'}{x} = \dfrac{e^{x^2}}{y}$.

37. Find the particular solution of the differential equation $\dfrac{dy}{dx} = 500 - y$ that satisfies the initial condition $y(0) = 7$.

Chapter 5 Integral Review Answer Key

7) $\int_1^{5e} \frac{1}{x}\,dx = \ln|x|\Big]_1^{5e} = \ln(5e) - \ln 1 = \ln 5 + \ln e - \ln 1 = \ln 5 + 1$ $\boxed{d}$

2) $\int \frac{x+2}{x+1}\,dx = \int \frac{u+1}{u}\,du = \int du + \int \frac{1}{u}\,du = u + \ln|u| + C = x+1+\ln|x+1| + C$

$u = x+1,\ du = dx$ $\quad = x + \ln|x+1| + C$ $\boxed{d}$

15) $\int x\cot x^2\,dx = \frac{1}{2}\int \cot u\,du = \frac{1}{2}\ln|\sin u| + C = \frac{1}{2}\ln|\sin x^2| + C$ $\boxed{d}$

$u = x^2,\ du = 2x\,dx$

16) $\int \frac{\cos^3 x - \sin^2 x}{\cos^2 x}\,dx = \int \frac{\cos^3 x - (1-\cos^2 x)}{\cos^2 x}\,dx = \int \frac{\cos^3 x + \cos^2 x - 1}{\cos^2 x}\,dx$

$= \int \cos x\,dx + \int dx - \int \sec^2 x\,dx = \sin x + x - \tan x + C$ $\boxed{c}$

17) $\int \sec 2x\,dx = \frac{1}{2}\int \sec u\,du = \frac{1}{2}\ln|\sec u + \tan u| + C = \boxed{\frac{1}{2}\ln|\sec 2x + \tan 2x| + C}$

$u = 2x,\ du = 2\,dx$

19) $\int \frac{\ln\sqrt{x}}{x}\,dx = \int \frac{\ln x^{\frac{1}{2}}}{x}\,dx = \int \frac{\frac{1}{2}\ln x}{x}\,dx = \int \frac{1}{2}u\,du = \frac{1}{4}u^2 + C = \boxed{\frac{1}{4}(\ln x)^2 + C}$

$u = \ln x,\ du = \frac{1}{x}\,dx$

21) $\int \frac{1-\sin\theta}{\cos\theta}\,d\theta = \int \sec\theta\,d\theta - \int \tan\theta\,d\theta = \boxed{\ln|\sec\theta + \tan\theta| + \ln|\cos\theta| + C}$

23) $\frac{dy}{dx} = \frac{3x}{1-x^2},\ \int dy = \int \frac{3x}{1-x^2}\,dx,\ y = -\frac{3}{2}\int \frac{1}{u}\,du = -\frac{3}{2}\ln|u| + C,\ \boxed{y = -\frac{3}{2}\ln|1-x^2| + C}$

$u = 1-x^2,\ du = -2x\,dx$

1) $\int \frac{x+3}{x^2+9}\,dx = \int \frac{x}{x^2+9}\,dx + \int \frac{3}{x^2+9}\,dx = \frac{1}{2}\int \frac{1}{u}\,du + 3\int \frac{1}{x^2+9}\,dx$

$u = x^2+9$
$du = 2x\,dx$

$= \frac{1}{2}\ln|u| + \frac{3}{3}\arctan\frac{x}{3} + C$ $\boxed{c}$

$= \frac{1}{2}\ln(x^2+9) + \arctan\frac{x}{3} + C$

2) $\int \frac{dx}{\sqrt{8+2x-x^2}} = \int \frac{dx}{\sqrt{9-(x-1)^2}} = \int \frac{du}{\sqrt{9-u^2}} = \arcsin\frac{u}{3} + C = \boxed{\arcsin\frac{x-1}{3} + C}$

(x^2-2x-8)

$u = x-1$
$du = dx$

$= -(x^2-2x+1-8-1)$

$= -((x-1)^2-9) = (9-(x-1)^2)$

3) $\int \frac{x+2}{\sqrt{4-x^2}}\,dx = \int \frac{x}{\sqrt{4-x^2}}\,dx + \int \frac{2}{\sqrt{4-x^2}}\,dx = -\frac{1}{2}\int u^{-\frac{1}{2}}du + 2\int \frac{1}{\sqrt{4-x^2}}\,dx$ $\boxed{B}$

$u = 4-x^2$
$du = -2x\,dx$

$= -u^{\frac{1}{2}} + 2\arcsin\frac{x}{2} + C = -\sqrt{4-x^2} + 2\arcsin\frac{x}{2} + C$

4) $\int \frac{5}{x^2+6x+13}\,dx = 5\int \frac{1}{(x+3)^2+4}\,dx = 5\int \frac{1}{u^2+4}\,dx = \frac{5}{2}\arctan\frac{u}{2} + C = \frac{5}{2}\arctan\frac{x+3}{2} + C$

$x^2+6x+9+13-9$
$= (x+3)^2+4$

$u = x+3$
$du = dx$

$\boxed{C}$

5) $\int \frac{x}{16+x^4}\,dx = \frac{1}{2}\int \frac{1}{16+u^2}\,du = \frac{1}{2}\cdot\frac{1}{4}\arctan\frac{u}{4} + C = \frac{1}{8}\arctan\frac{x^2}{4} + C$ $\boxed{B}$

$u = x^2$, $du = 2x\,dx$

7) $\int_1^{\sqrt{e}} \frac{-4x}{x^2}\,dx = \int_1^{\sqrt{e}} -\frac{4}{x}\,dx = -4\ln|x|\Big]_1^{\sqrt{e}} = -4\ln\sqrt{e} - -4\ln 1 = -4(\ln e^{\frac{1}{2}} - \ln 1)$ $\boxed{D}$

$= -4(\frac{1}{2} - 0) = -2$

10) $\int \frac{1}{2x\sqrt{4x^2-1}}\,dx = \int \frac{1}{2x(2)\sqrt{x^2-\frac{1}{4}}}\,dx = \frac{1}{4}\int \frac{1}{x\sqrt{x^2-\frac{1}{4}}} = \frac{1}{4}\cdot 2\,\mathrm{arcsec}\frac{|x|}{\frac{1}{2}} + C = \frac{1}{2}\mathrm{arcsec}\,2|x| + C$

$\boxed{D}$

10) $\int \frac{3x^2+3x+3}{x^2+1}\,dx = 3\int \frac{x^2+x+1}{x^2+1}\,dx = 3\int \frac{x^2+1}{x^2+1}\,dx + 3\int \frac{x}{x^2+1} = 3\int dx + \frac{3}{2}\int \frac{1}{u}\,du$

$u = x^2+1$
$du = 2x\,dx$

$= 3x + \frac{3}{2}\ln(x^2+1) + C$

$\boxed{C}$

17) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$ $\boxed{A}$

$u = \sqrt{x}$, $du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}\,dx$

21) $\int \frac{1}{x^2 e^{5/x}}\,dx = \frac{1}{5}\int e^u\,du = \frac{1}{5}e^u + C = \frac{1}{5}e^{-\frac{5}{x}} + C$ $\boxed{C}$

$u = -\frac{5}{x}$, $du = 5x^{-2}dx$ $\frac{1}{5}du = \frac{1}{x^2}dx$

23) $\int_1^4 e^{2x}\,dx = \frac{1}{2}\int_2^8 e^u\,du = \frac{1}{2}e^u\Big]_2^8 = \boxed{\frac{1}{2}(e^8 - e^2)}$

$u = 2x$, $du = 2\,dx$

24) $f(x) = \ln(e^{-x^2}) = -x^2\ln e = -x^2$ $f'(x) = -2x$ $\boxed{C}$

17) $\int x\,3^{x^2}\,dx = \frac{1}{2}\int 3^u\,du = \frac{1}{2}\cdot\frac{3^u}{\ln 3} + C = \frac{3^{x^2}}{2\ln 3} + C$ $\boxed{C}$

$u = x^2$ $du = 2x\,dx$

12) $\frac{dR}{dt} = kR \rightarrow \int \frac{1}{R}\,dR = \int k\,dt \rightarrow \ln R = kt + C \rightarrow R = Ce^{kt}$

$\frac{1}{2}C = Ce^{50k} \rightarrow \frac{1}{2} = e^{50k} \rightarrow \ln\left(\frac{1}{2}\right) = 50k \rightarrow k = \frac{1}{50}\ln\left(\frac{1}{2}\right) \qquad R = Ce^{\frac{1}{50}\ln\left(\frac{1}{2}\right)t}$

$R = e^{\frac{1}{50}\ln\left(\frac{1}{2}\right)(60)} \approx .43528 \rightarrow 43.53\% \quad \boxed{A}$

12) $\displaystyle\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{3}{\sqrt{4-9x^2}}\,dx = \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{3}{3\sqrt{\frac{4}{9}-x^2}}\,dx = \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{1}{\sqrt{\frac{4}{9}-x^2}}\,dx = \arcsin \frac{3x}{2}\Big]_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}$

$= \arcsin\frac{3}{2\sqrt{3}} - \arcsin\frac{-3}{2\sqrt{3}} = \arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{-\sqrt{3}}{2} = \frac{\pi}{3} - -\frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

25) $\displaystyle\int \frac{1}{(x-3)\sqrt{x^2-6x+5}}\,dx = \int \frac{1}{(x-3)\sqrt{(x-3)^2-4}}\,dx = \int \frac{1}{u\sqrt{u^2-4}} = \frac{1}{2}\operatorname{arcsec}\frac{|u|}{2} + C$

x^2-6x+5
$= x^2-6x+9+5-9$
$= (x-3)^2-4$

$u = x-3$
$du = dx$

$= \boxed{\frac{1}{2}\operatorname{arcsec}\frac{|x-3|}{2} + C}$

2) $A = Pe^{rt} \qquad 3000 = 1000e^{.05t} \rightarrow \ln 3 = .05t$

$A = 1000e^{.05t} \qquad 3 = e^{.05t} \qquad t = \frac{\ln 3}{.05} \approx 21.97$ years $\boxed{D}$

6) $\int y' = \int \frac{2}{\sqrt{1-x^2}} + x \qquad y = 2\arcsin x + \frac{1}{2}x^2 + C \quad \boxed{B}$

7) $\int y' = \int x\sqrt{x+1} \qquad y = \int (u-1)\cdot u^{\frac{1}{2}}\,du = \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du = \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C$

$u = x+1$
$du = dx$

$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C \quad \boxed{E}$

$= (x+1)^{\frac{3}{2}}\left[\frac{2}{5}(x+1) - \frac{2}{3}\right] + C$

19) $\sqrt{x^2-1}\,\frac{dy}{dx} = \frac{y}{x} \rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x\sqrt{x^2-1}}\,dx \rightarrow \ln|y| = \operatorname{arcsec} x + C \rightarrow \boxed{y = Ce^{\operatorname{arcsec} x}}$

21) a) $\frac{dy}{dx} = \frac{k}{\sqrt{y}}$ b) $\int \sqrt{y}\,dy = \int k\,dx \rightarrow \frac{2}{3}y^{\frac{3}{2}} = kx + C \rightarrow y^{\frac{3}{2}} = \frac{3}{2}kx + C \rightarrow \boxed{y = \left(\frac{3}{2}kx\right)^{\frac{2}{3}} + C}$

24) $\frac{df}{dt} = kf \rightarrow \int \frac{1}{f}\,df = \int k\,dt \rightarrow \ln|f| = kt + C \rightarrow f = Ce^{kt} \rightarrow 10 = Ce^0 = C \rightarrow f = 10e^{kt}$

$24 = 10e^{6k} \rightarrow \frac{12}{5} = e^{6k} \rightarrow \ln\frac{12}{5} = 6k \rightarrow k = \frac{1}{6}\ln\frac{12}{5} \rightarrow f = 10e^{\frac{1}{6}\ln\frac{12}{5}t} \quad \boxed{A}$

$\frac{dy}{dx} = \frac{e^{x^2}}{y} \rightarrow \int y\,dy = \int xe^{x^2}dx \rightarrow \frac{1}{2}y^2 = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C \rightarrow \boxed{y^2 = e^{x^2} + C}$

$u = x^2,\ du = 2x\,dx$

37) $\frac{dy}{dx} = 500 - y \rightarrow \frac{1}{500-y}\,dy = dx \rightarrow \int -\frac{1}{u}\,du = \int dx \rightarrow -\ln|u| = x + C \rightarrow -\ln|500-y| = x + C \rightarrow \frac{1}{500-y} = Ce^x$

$500 - y = Ce^{-x} \rightarrow y = 500 - Ce^{-x}$

$u = 500-y,\ du = -dy$

$7 = 500 - Ce^0 \rightarrow C = 493 \rightarrow \boxed{y = 500 - \frac{493}{e^x}}$