A.P. Calculus AB | **Free Response Review for Test On Integrals of Logs/Exps**

Calculators permitted.

Answer Key
Name

1. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining.

t	y
0	1000000
6	500000

a) Write an equation for y, the amount of oil remaining in the well at any time t.

$\int \frac{1}{y}dy = \int k\,dt$

$\ln y = kt + C$

$y = e^{kt+C}$

$y = Ae^{kt}$

$1000000 = Ae^0 = A$

$y = 1000000e^{kt}$

$500000 = 1000000e^{6k}$

$\frac{1}{2} = e^{6k}$

$\ln\frac{1}{2} = 6k$

$k = \frac{1}{6}\ln\frac{1}{2}$

$\boxed{y = 1{,}000{,}000e^{\left(\frac{1}{6}\ln\frac{1}{2}\right)t}}$

b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

$\frac{dy}{dt} = (600000)\left(\frac{1}{6}\ln\frac{1}{2}\right)$

$\approx -69314.718 \frac{\text{gallons}}{\text{year}}$

b) It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining. In order not to lose money, at what time t should oil no longer be pumped from the well?

t	y
?	50000

$50000 = 1000000\,e^{\left(\frac{1}{6}\ln\frac{1}{2}\right)t}$

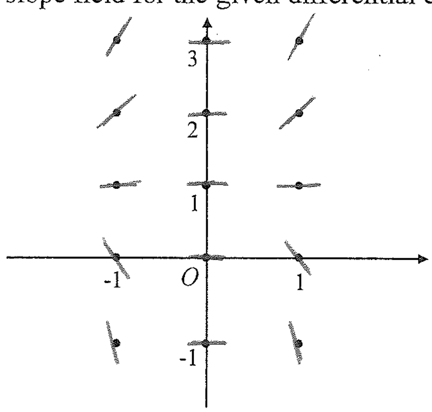
$\frac{1}{20} = e^{\left(\frac{1}{6}\ln\frac{1}{2}\right)t}$

$\ln\left(\frac{1}{20}\right) = \left(\frac{1}{6}\ln\frac{1}{2}\right)t$

$t = \frac{6\ln\left(\frac{1}{20}\right)}{\ln\left(\frac{1}{2}\right)} \approx 25.932$ years

2. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

a) On the axes below, sketch a slope field for the given differential equation at the fifteen points indicated.



b) While the slope field in part a is drawn at only fifteen points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

$x^2(y-1) < 0$ $\quad$ $y < 1,\ x \neq 0$

c) Find the particular solution, $y = f(x)$ to the given diff. equation with the initial condition $f(0) = 3$.

$\int \frac{1}{y-1}dy = \int x^2dx$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$|y-1| = e^{\frac{1}{3}x^3+C} = Ae^{\frac{1}{3}x^3}$

$y-1 = Me^{\frac{1}{3}x^3}$

$y = Me^{\frac{1}{3}x^3} + 1$

$3 = Me^0 + 1 \Rightarrow M = 2$

$y = 2e^{\frac{1}{3}x^3} + 1$

d) Find the equation of the line tangent to $y = f(x)$ at the point where $x = 0$ and use it to approximate $f(0.1)$.

point: $(0, 3)$

slope: $\frac{dy}{dx} = 0$

$y - 3 = 0(x-0)$

$y = 3$

$f(0.1) \approx 3$

3. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2(v+16)$, with initial condition $v(0) = -50$.

a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

$$\int \frac{1}{v+16}\,dv = \int -2\,dt$$

$$\ln|v+16| = -2t + C$$

$$|v+16| = e^{-2t+C} = Ae^{-2t}$$

$$v+16 = Me^{-2t}$$

$$v = Me^{-2t} - 16$$

$$-50 = Me^{0} - 16$$

$$M = -34$$

$$v = -34e^{-2t} - 16$$

b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

$$\lim_{t\to\infty} -34e^{-2t} - 16 = \lim_{t\to\infty} \frac{-34}{e^{2t}} - 16$$

$$= 0 - 16 = -16 \frac{ft}{sec}$$

c) It is safe to land when her <u>speed</u> is 20 feet per second. At what time, t, does she reach this speed?

falling, so when speed $= 20$, $v = -20$

$$-20 = -34e^{-2t} - 16$$

$$-4 = -34e^{-2t}$$

$$\frac{4}{34} = e^{-2t}$$

$$\ln\frac{4}{34} = -2t$$

$$t = -\frac{1}{2}\ln\frac{4}{34}$$

$$\approx 1.070 \text{ seconds}$$