Review of Tangent Lines - Answer Key

1969 #20

$y = \arcsin \frac{1}{2}x$

$y' = \frac{1}{\sqrt{1-(\frac{1}{2}x)^2}} \cdot \frac{1}{2}$ $\quad y'(0) = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x-0)$

$y = \frac{1}{2}x \quad x - 2y = 0$ $\boxed{A}$

1969 #36

$y = \sqrt{4+\sin x}$

$y' = \frac{1}{2}(4+\sin x)^{-\frac{1}{2}}(\cos x)$

$= \frac{\cos x}{2\sqrt{4+\sin x}}$

$y(0) = \sqrt{4} = 2$

$y'(0) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\boxed{B}$

$y - 2 = \frac{1}{4}(x-0)$

at $x = .12$

$y - 2 = \frac{1}{4}(.12) = .03$

$y = 2.03$

1973 #3

$y = \ln(x^2)$

$y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$ $\quad y'(e^2) = \frac{2}{e^2}$ $\quad \boxed{B}$

1973 #11

$3x - 4y = 0$

$4y = 3x$

$y = \frac{3}{4}x$ so slope $= \frac{3}{4}$

$y = x^3 + K$

$y' = 3x^2 = \frac{3}{4}$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

$y(\frac{1}{2}) = \frac{1}{8} + K$

$\frac{3}{4}(\frac{1}{2}) = \frac{1}{8} + K$

$\frac{3}{8} = \frac{1}{8} + K$

$K = \frac{1}{4}$

$\boxed{B}$

1985 #8

$y = \ln(\frac{1}{2}x)$

$y' = \frac{1}{\frac{1}{2}x} \cdot \frac{1}{2} = \frac{1}{x}$

$y'(4) = \frac{1}{4}$ $\quad \boxed{B}$

1985 #43

$y = x^3 + 3x^2 + 2$

$y' = 3x^2 + 6x$

$y'' = 6x + 6 = 0$

at $x = -1$

$y(-1) = 4$

$y'(-1) = -3$

$y - 4 = -3(x - -1)$

$y - 4 = -3x - 3$

$y = -3x + 1$

$\boxed{B}$

1988 #11

$f(x) = x(1-2x)^3$

$f'(x) = (1-2x)^3 + x(3)(1-2x)^2(-2)$

$f'(1) = -1 + 3(-2) = -7$

$y - -1 = -7(x-1)$

$y + 1 = -7x + 7$

$y = -7x + 6$

$\boxed{A}$

1993 #7

$y = \frac{2x+3}{3x-2}$

$y' = \frac{2(3x-2) - (2x+3)(3)}{(3x-2)^2}$

$y'(1) = \frac{2(1) - 5(3)}{1}$

$= -13$

$y - 5 = -13(x-1)$

$y - 5 = -13x + 13$

$13x + y = 18$

$\boxed{B}$

1997 #10

$y = \cos(2x)$

$y' = -\sin(2x) \cdot 2$

$y'(\frac{\pi}{4}) = -2\sin(\frac{\pi}{2}) = -2$

$y(\frac{\pi}{4}) = \cos(\frac{\pi}{2}) = 0$

$y - 0 = -2(x - \frac{\pi}{4})$

$\boxed{E}$

1997 #12

$2x-4y=3$ $\quad y=\frac{1}{2}x^2$

$4y=2x-3$ $\quad y'=x=\frac{1}{2}$

$y=\frac{1}{2}x-\frac{3}{4}$ $\quad y\left(\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1}{2}\right)^2=\frac{1}{8}$ $\quad \boxed{B}$

1997 #14 $\quad f(3)=2\ ,\ f'(3)=5$

$y-2=5(x-3)$

$y-2=5x-15 \rightarrow y=5x-13=0$

$x=\frac{13}{5}=2.6$ $\quad \boxed{C}$

1997 #80 $\quad f(x)=2e^{4x^2}$

$f'(x)=2e^{4x^2}\cdot 8x=16xe^{4x^2}=3$ $\quad x=.168$ $\quad \boxed{A}$

1998 #18 $\quad y=x+\cos x \rightarrow y'(0)=1$

$y'=1-\sin x$ $\quad y-1=1(x-0) \rightarrow y=x+1$ $\quad \boxed{B}$

1998 #77 $\quad f(x)=3e^{2x}$ $\quad g(x)=6x^3$ $\quad 6e^{2x}=18x^2$

$f'(x)=6e^{2x}$ $\quad g'(x)=18x^2$ $\quad x\approx -.391$ $\quad \boxed{C}$

1998 #87 $\quad f(x)=x^4+2x^2$ $\quad f(.237)\approx .115$

$f'(x)=4x^3+4x=1$ $\quad y-.115=1(x-.237) \rightarrow y=x-.122$

$x\approx .237$ $\quad \boxed{D}$

2003 #24 $\quad f(x)=4x^3-5x+3 \rightarrow f(-1)=4$ $\quad y-4=7(x-^-1)$

$f'(x)=12x^2-5 \longrightarrow f'(-1)=7$ $\quad y-4=7x+7 \rightarrow y=7x+11$ $\quad \boxed{C}$

2003 #26 $\quad 3y^2-2x^2=6-2xy$

$6y\frac{dy}{dx}-4x=-2y-2x\frac{dy}{dx} \rightarrow 6(2)\frac{dy}{dx}-4(3)=-2(2)-2(3)\frac{dy}{dx}$

$12\frac{dy}{dx}-12=-4-6\frac{dy}{dx}$

$18\frac{dy}{dx}=8$ $\quad \frac{dy}{dx}=\frac{4}{9}$ $\quad \boxed{B}$

2003 #89 $\quad f(2)=3$ $\quad f'(2)=-5$ $\quad g'(x)=f(x)+xf'(x)$ $\quad y-6=-7(x-2)$

$g(x)=xf(x)$ $\quad g(2)=2(3)=6$ $\quad g'(2)=f(2)+2f'(2)$

$=3+2(-5)=-7$ $\quad \boxed{D}$

1989 #1 $f(x)=x^3-7x+6$

a) $x=\pm 1,2,3,6$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$f(1)=0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3,2$

zeros are $1,2,-3$

b) $f'(x)=3x^2-7$ $f(-1)=12$

$f'(-1)=-4$

$y-12=-4(x+1)$

c) slope of secant $=\dfrac{f(3)-f(1)}{3-1}=\dfrac{27-21+6-0}{2}=6$

$f'(x)=3x^2-7=6$

$3x^2=13 \rightarrow x^2=\frac{13}{3}$ $x=\pm\sqrt{\frac{13}{3}}$ $c=\sqrt{\frac{13}{3}}$

1991 #1 $f''(x)=24x-18$

a) $f'(x)=12x^2-18x+C$

$f'(1)=12-18+C=-6$

$C=0$

$f'(x)=12x^2-18x=0$

$6x(2x-3)=0$

$x=0,\frac{3}{2}$

b) $f'(x)=12x^2-18x$

$f(x)=4x^3-9x^2+K \rightarrow f(2)=32-36+K=0$

$K=4$

$f(x)=4x^3-9x^2+4$

c) $\dfrac{1}{3-1}\displaystyle\int_1^3(4x^3-9x^2+4)dx=\frac{1}{2}\left(x^4-3x^3+4x\Big]_1^3\right)=\frac{1}{2}\left(81-81+12-(1-3+4)\right)$

$=\frac{1}{2}(12-2)=5$

1991 #3 $f(x)=(1+\tan x)^{3/2} \rightarrow f(0)=1$

a) $f'(x)=\frac{3}{2}(1+\tan x)^{\frac{1}{2}}(\sec^2 x)\rightarrow f'(0)=\frac{3}{2}(1)(1)=\frac{3}{2}$

$y-1=\frac{3}{2}(x-0)$

$y=\frac{3}{2}x+1$

b) $y=\frac{3}{2}(.02)+1=\frac{3}{2}\left(\frac{2}{100}\right)+1=\frac{3}{100}+1=\frac{103}{100}$ or 1.03

c) $x=(1+\tan y)^{\frac{3}{2}}$

$x^{\frac{2}{3}}=1+\tan y \rightarrow \tan y=x^{\frac{2}{3}}-1$

$y=\arctan\left(x^{\frac{2}{3}}-1\right)$

$f^{-1}(x)=\arctan\left(x^{\frac{2}{3}}-1\right)$

1992 #4 $y+\cos y=x+1$

a) $\frac{dy}{dx}-\sin y\frac{dy}{dx}=1$

$\frac{dy}{dx}(1-\sin y)=1 \rightarrow \frac{dy}{dx}=\dfrac{1}{1-\sin y}$

b) $1-\sin y=0$

$\sin y=1 \rightarrow y=\frac{\pi}{2}$

$\frac{\pi}{2}+\cos\frac{\pi}{2}=x+1 \rightarrow \underline{x=\frac{\pi}{2}-1}$

c) $\frac{dy}{dx}=(1-\sin y)^{-1}$ $\frac{d^2y}{dx^2}=-(1-\sin y)^{-2}\left(-\cos y\frac{dy}{dx}\right)=\dfrac{\cos y\left(\frac{1}{1-\sin y}\right)}{(1-\sin y)^2}$

$=\dfrac{\cos y}{(1-\sin y)^3}$

1994 #3 $x^2+xy+y^2=27$

a) $2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx}=0 \rightarrow \frac{dy}{dx}(x+2y)=-2x-y$

$x\frac{dy}{dx}+2y\frac{dy}{dx}=-2x-y$ $\quad \frac{dy}{dx}=\frac{-2x-y}{x+2y}$

b) $x^2=27$ (when $y=0$) $\quad \frac{dy}{dx}(\sqrt{27},0)=\frac{-2\sqrt{27}}{\sqrt{27}}=-2 \quad \frac{dy}{dx}(-\sqrt{27},0)=\frac{2\sqrt{27}}{-\sqrt{27}}=-2$

$x=\pm\sqrt{27}=\pm 3\sqrt{3}$

yes, they are parallel because the slope is -2 at both points

c) $x+2y=0 \quad (-2y)^2-2y(y)+y^2=27$

$x=-2y \quad 4y^2-2y^2+y^2=27$

$3y^2=27 \rightarrow y^2=9 \rightarrow y=\pm 3$

$x=-2(3)=-6$

$x=-2(-3)=6 \quad (-6,3),(6,-3)$

1995 #3 $-8x^2+5xy+y^3=-149$

a) $-16x+5y+5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0 \rightarrow \frac{dy}{dx}(5x+3y^2)=16x-5y$

$5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=16x-5y \quad \frac{dy}{dx}=\frac{16x-5y}{5x+3y^2}$

b) $\frac{dy}{dx}(4,-1)=\frac{64+5}{20+3}=\frac{69}{23}=3 \quad y+1=3(x-4)$

c) $y+1=3(4.2-4) \rightarrow y+1=3(.2) \rightarrow y+1=.6 \rightarrow y=-.4$

d) $-8(4.2)^2+5(4.2)K+K^3=-149$

e) $-141.12+21K+K^3=-149 \rightarrow K\approx -.373$ (on calculator)

1996 #6 $y=x-\frac{1}{500}x^2 \rightarrow \frac{dy}{dx}=1-\frac{1}{250}x$ let Q be (a,b)

a) (there are several methods for this) tangent line is $y-20=(1-\frac{1}{250}x)(x-0)$ substitute for y to get

$x-\frac{1}{500}x^2-20=(1-\frac{1}{250}x)(x)$

solve to get $x=100$

b) $y=(1-\frac{1}{250}(100))x+20$

$y=\frac{3}{5}x+20$

c) vertex is at $x=250$, so $y=250-\frac{1}{500}(250^2)=125$

adding 50 feet the tree has a highest point of 175 feet

at $x=250$ the y-value on l is $\frac{3}{5}(250)+20=170$

yes, the light will hit the tree 45 feet up

1997 #2

a) slope $= \frac{0-3}{\frac{\pi}{2}-0} = \frac{-3}{\frac{\pi}{2}} = \frac{-6}{\pi}$ $\quad y-3 = \frac{-6}{\pi}(x-0)$ or $y = \frac{-6}{\pi}x+3$

b) $f(x) = 3\cos x$ $\quad f'(x) = -3\sin x$ $\quad f'(\frac{\pi}{2}) = -3\sin\frac{\pi}{2} = -3$

$y-0 = -3(x-\frac{\pi}{2})$ or $y = -3x+\frac{3\pi}{2}$

c) $-3\sin x = -\frac{6}{\pi} \rightarrow \sin x = \frac{2}{\pi} \rightarrow x \approx .690$

d)  $V = \pi \int_0^{\frac{\pi}{2}} (3\cos x - 0)^2 - (-\frac{6}{\pi}x+3-0)^2 \, dx$

2001 #4 $\quad h'(x) = \frac{x^2-2}{x}$ $\quad x^2-2=0$, $x = \pm\sqrt{2}$ $\quad h'(x)$: $-$ $+$ $-$ $+$

x: $\searrow -\sqrt{2} \nearrow 0 \searrow \sqrt{2} \nearrow$

a) rel. min. at $x = \pm\sqrt{2}$ b/c $h'(x)$ chgs from (+) to (-)

b) $h''(x) = \frac{2x(x)-(x^2-2)}{x^2} = \frac{x^2+2}{x^2}$ $\quad x^2+2 \neq 0$ $\quad$ $+$ 0 $+$ $\quad$ no poi. b/c $h''(x)$ does not chg sign

c) $h'(4) = \frac{14}{4} = \frac{7}{2}$ $\quad h(4) = -3$ $\quad y+3 = \frac{7}{2}(x-4)$

d) it lies below the graph b/c $h(x)$ is concave up on $x > 4$

2002 #6

a) $\int_0^{1.5} (3f'(x)+4)\,dx = 3f(x)+4x\Big]_0^{1.5} = 3f(1.5)+4(1.5)-(3f(0)-4(0))$
$= 3(-1)+6-3(-7) = 24$

b) point $(1,-4)$ slope $= f'(1) = 5 \Rightarrow y+4 = 5(x-1) \rightarrow y+4 = 5(1.2-1) \rightarrow y+4 = 1 \rightarrow y = -3$
This would be less than $f(1.2)$ because $f(x)$ is concave up.

c) Mean Value Theorem: $\frac{f'(.5)-f'(0)}{.5-0} = \frac{3-0}{.5} = 6 \rightarrow r = 6$ by the Mean Value Thm

d) $g'(x) = \begin{cases} 4x-1, & x<0 \\ 4x+1, & x>0 \end{cases}$ They cannot be the same function because $f'(0) = 0$ but $g'(0)$ is undefined

2006 #6

a) $g(x) = e^{ax}+f(x)$ so $g'(x) = ae^{ax}+f'(x) \rightarrow g'(0) = a+f'(0) = a-4$
$g''(x) = a^2e^{ax}+f''(x) \rightarrow g''(0) = a^2+f''(0) = a^2+3$

b) $h(x) = \cos(Kx)\cdot f(x)$
$h'(x) = -K\sin(Kx)\cdot f(x) + \cos(Kx)\cdot f'(x)$
$h(0) = \cos(0)\cdot f(0) = 1\cdot 2 = 2 \rightarrow$ point $(0,2)$
$h'(0) = -K\sin(0)\cdot f(0) + \cos(0)\cdot f'(0)$
$= 0 + 1\cdot -4 = -4 \rightarrow$ slope $= -4$

$y-2 = -4(x-0)$
or $y-2 = -4x$
or $y = -4x+2$