# Review of PVA Problems

1969 #19

$v(t) = \frac{\ln t}{t}$ $\quad v'(t) = \frac{\frac{1}{t}\cdot t - \ln t}{t^2} = \frac{1-\ln t}{t^2}$ $\quad 1-\ln t = 0$, $\ln t = 1$, $t = e$ $\quad t^2 = 0$, $t = 0$ but $t > 0$

$v'(t)$: $+$ on $(0, e)$, $-$ after $e$; $t$: $0 \nearrow e \searrow$ $\quad \boxed{C}$

1969 #35

$a(t) = 24t^2$ so $v(t) = 8t^3 + C$, $v(0) = 0 + C = 0$ so $\rightarrow v(t) = 8t^3$ $\quad \int_0^2 8t^3\,dt = 2t^4\Big]_0^2 = 32$ $\quad \boxed{A}$

1973 #8

$v(t) = t^2$ $\quad \int_1^2 t^2\,dt = \frac{1}{3}t^3\Big]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ $\quad \boxed{B}$

1973 #13

$a(t) = 8-6t$ $\rightarrow v(t) = 8t - 3t^2 + C$, $v(1) = 8 - 3 + C = 25$ $\rightarrow C = 20$, $v(t) = 8t - 3t^2 + 20$

$\int_2^4 (8t - 3t^2 + 20)\,dt = 4t^2 - t^3 + 20t\Big]_2^4 = 64 - 64 + 80 - (16 - 8 + 40) = 32$ $\quad \boxed{D}$

1973 #28

$d(t) = 8t - 3t^2$ $\rightarrow v(t) = 8 - 6t = 0$, $t = \frac{4}{3}$ $\quad d(1) = 5$, $d\left(\frac{4}{3}\right) = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ $\quad d(2) = 4$

total distance $= \frac{16}{3} - 5 + \frac{16}{3} - 4 = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$ $\quad \boxed{C}$

1985 #11 $\quad s(t) = t^2 + 4t + 4 \rightarrow v(t) = 2t + 4 \rightarrow a(t) = 2$ $\quad \boxed{B}$

1985 #14 $\quad v(t) = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}} = 0$, $t^{\frac{1}{2}}(3 + 5t) = 0$ $\rightarrow t = 0, t = -\frac{3}{5}$

$\int_0^4 \left(3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}\right)dt = 2t^{\frac{3}{2}} + 2t^{\frac{5}{2}}\Big]_0^4 = 16 + 64 = 80$ $\quad \boxed{D}$

1985 #28 $\quad x(t) = -5t^2 \rightarrow v(t) = -10t$

$\frac{1}{3-0}\int_0^3 -10t\,dt = \frac{1}{3}\left(-5t^2\Big]_0^3\right) = \frac{1}{3}(-45 - 0) = -15$ $\quad \boxed{C}$

1988 #3 $\quad v(t) = e^t = 0$ <u>never!</u> $\quad \int_0^2 e^t\,dt = e^t\Big]_0^2 = e^2 - 1$ $\quad \boxed{A}$

1988 #16 $\quad x(t) = t^3 - 3t^2 - 9t + 1$ $\quad v(t) = 3t^2 - 6t - 9 = 0$, $t^2 - 2t - 3 = 0$, $(t-3)(t+1) = 0$ $\quad t = 3, \cancel{-1}$ $\quad \boxed{C}$

1993 #11 $\quad a(t) = 6t - 2$

$v(t) = 3t^2 - 2t + C$, $v(3) = 27 - 6 + C = 25$, $C = 4$

$v(t) = 3t^2 - 2t + 4$, $x(t) = t^3 - t^2 + 4t + K$, $x(1) = 1 - 1 + 4 + K = 10$, $K = 6$ $\quad \boxed{C}$

1993 #26 $s(t) = -4\cos t - \frac{t^2}{2} + 10$ → $\cos t = \frac{1}{4}$

$v(t) = 4\sin t - t$ $a(t) = 4\cos t - 1 = 0$ $t \approx 1.318$ $\boxed{D}$

$v(1.318) \approx 2.555$

1997 #8 chgs dir. when $v(t)$ chgs sign at $t=6$ $\boxed{C}$

1997 #9 $\frac{1}{2}(2)(3) + 2(3) + \frac{1}{2}(2)(3) + \frac{1}{2}(2)(1) = 13$ $\boxed{B}$

1997 #87 $a(t) = t + \sin t$ → $v(0) = 0 - 1 + C = -2$

$v(t) = \frac{1}{2}t^2 - \cos t + C$ $C = -1$ $\boxed{B}$

$v(t) = \frac{1}{2}t^2 - \cos t - 1 = 0$

$t \approx 1.478$

1998 #14 $x(t) = t^2 - 6t + 5$

$v(t) = 2t - 6 = 0$ at $t = 3$ $\boxed{C}$

1998 #24 $v(t) = t^3 - 3t^2 + 12t + 4$ → $a'(t) = 6t - 6 = 0$

$a(t) = 3t^2 - 6t + 12$ at $t = 1$ $\boxed{A}$

$a(0) = 12$, $a(1) = 9$, $a(3) = 21$

2003 #25 $x(t) = 2t^3 - 21t^2 + 72t - 53$ → $(t-4)(t-3) = 0$

$v(t) = 6t^2 - 42t + 72 = 0$ $t = 4, 3$ $\boxed{E}$

$t^2 - 7t + 12 = 0$

2003 #76

$v(t) = 3 + 4.1\cos(0.9t)$ $v'(4) \approx 1.633$ $\boxed{C}$

2003 #83 $v(t) = e^t + te^t$ $\frac{1}{3-0}\int_0^3 v(t)\,dt \approx 20.086$ $\boxed{A}$

2003 #91 $a(t) = \ln(1 + 2^t)$ $\int_1^2 a(t)\,dt = v(t)\Big]_1^2 = v(2) - v(1)$

$1.346 = v(2) - 2$ $\boxed{E}$

so $v(2) \approx 3.346$

1989 #3 $a(t) = 4\cos(2t)$

a) $u = 2t$ $\quad \int 4\cos(2t)dt = \int 2\cos(u)du = 2\sin u + C = 2\sin(2t) + C$

$du = 2dt$ $\quad v(0) = 2\sin(0) + C = 1 \quad v(t) = 2\sin(2t) + 1$

$\frac{1}{2}du = dt$ $\quad C = 1$

b) $x(t) = \int (2\sin(2t) + 1)dt = -\cos(2t) + t + K$

$x(0) = -\cos 0 + 0 + K = 0$ $\quad x(t) = -\cos(2t) + t + 1$

$-1 + K = 0$ so $K = 1$

c) $v(t) = 2\sin(2t) + 1 = 0 \quad \sin(2t) = -\frac{1}{2}$ so $t = \frac{7\pi}{12}, \frac{11\pi}{12}$

$2\sin(2t) = -1 \quad 2t = \frac{7\pi}{6}, \frac{11\pi}{6}$

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1992 #2

a) $v(t) = 3(t-1)(t-3)$ $\quad a(0) = -12$ so the min acceleration is $-12$

$a(t) = 3(t-3) + 3(t-1) = 6t - 12$ $\quad a(5) = 18$

$a'(t) = 6 \neq 0$ so no crit #s

b) $v(t) = 3(t^2 - 4t + 3) = 3t^2 - 12t + 9$ $\quad$ so $x(t) = t^3 - 6t^2 + 9t - 2$

$x(t) = t^3 - 6t^2 + 9t + C$ $\quad v(t) = 0$ at $t = 1, t = 3$

$x(2) = 8 - 24 + 18 + C = 0$

$C = -2$

$\int_0^1 v(t)dt = x(1) - x(0) = 2 - -2 = 4$ $\quad \int_1^3 v(t)dt = x(3) - x(1)$

$= 27 - 54 + 27 - 2 - 2 = -4$

$\int_3^5 v(t)dt = x(5) - x(3) =$

$= 125 - 150 + 45 - 2 - (-2) = 20$ $\quad 4 + 4 + 20 = 28$

c) $\frac{1}{5-0}\int_0^5 v(t)dt = \frac{1}{5}(x(5) - x(0)) = \frac{18 - -2}{5} = 4$

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1993 #2 $x(t) = 2te^{-t}$ $\quad \rightarrow a(0) = -4 + 0 = -4$

a) $v(t) = 2e^{-t} - 2te^{-t}$

$a(t) = -2e^{-t} - 2e^{-t} + 2te^{-t} = -4e^{-t} + 2te^{-t}$

b) $a(t) = -4e^{-t} + 2te^{-t} = -2e^{-t}(2 - t) = 0$ at $t = 2$

$v(2) = 2e^{-2} - 4e^{-2} = -2e^{-2}$ or $\frac{-2}{e^2}$

c) $v(t) = 2e^{-t} - 2te^{-t} = 2e^{-t}(1 - t) = 0$ at $t = 1$

$\int_0^1 v(t)dt = x(1) - x(0) = 2e^{-1} - 0 = \frac{2}{e}$ $\quad \int_1^5 v(t)dt = x(5) - x(1) = 10e^{-5} - 2e^{-1}$

$= \frac{10}{e^5} - \frac{2}{e} < 0$

total distance $= \frac{2}{e} + \frac{2}{e} - \frac{10}{e^5} = \boxed{\frac{4}{e} - \frac{10}{e^5}}$

1994 #4 $\quad v(t) = t\ln t - t$

a) $a(t) = v'(t) = \ln t + t\cdot\frac{1}{t} - 1 = \ln t + 1 - 1 = \ln t$

b) $v(t) = t\ln t - t = 0 \to t(\ln t - 1) = 0 \to t = 0$ or $\ln t = 1$

$t = e$

$v(t)$: $-$ on $(0, e)$, $+$ on $(e, \infty)$; $t$: $0$, $e$

the particle is moving right on $(e, \infty)$ b/c $v(t) > 0$

c) $a(t) = \ln t = 0$ at $t = 1$ $\quad a(t)$: $-$ on $(0,1)$ $\searrow$, $+$ after $1$ $\nearrow$; $t$: $0$, $1$

$v(1) = (1)\ln(1) - 1 = -1$

the min. velocity is $-1$ b/c $a(t)$ chgs from $(-)$ to $(+)$ at $t = 1$ and this is the only critical point (or b/c $a(t) < 0$ for all $t$ in $(0,1)$ and $a(t) > 0$ for all $t$ in $(1, \infty)$)

d) $v(t) = t\ln t - t = t(\ln t - 1)$

$u = \ln t - 1 \quad du = \frac{1}{t}\,dt$

$v = \frac{1}{2}t^2 \quad dv = t\,dt$

$x(t) = \int t(\ln t - 1)\,dt = \frac{1}{2}t^2(\ln t - 1) - \int \frac{1}{2}t^2\cdot\frac{1}{t}\,dt = \frac{1}{2}t^2(\ln t - 1) - \int \frac{1}{2}t\,dt$

$= \frac{1}{2}t^2(\ln t - 1) - \frac{1}{4}t^2 + C$

$x(1) = \frac{1}{2}(-1) - \frac{1}{4} + C = 6 \to -\frac{3}{4} + C = 6$

$C = 6\frac{3}{4} = \frac{27}{4}$

$x(t) = \frac{1}{2}t^2(\ln t - 1) - \frac{1}{4}t^2 + \frac{27}{4}$

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1995 #2 $\quad v(t) = t\cos t$

a) $t\cos t = 0$ at $t = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ $\quad v(t)$: $+$ on $(0, \frac{\pi}{2})$, $-$ on $(\frac{\pi}{2}, \frac{3\pi}{2})$, $+$ on $(\frac{3\pi}{2}, 5)$; $t$: $0$, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $5$

the particle is moving up on $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 5)$ b/c $v(t) > 0$

b) $a(t) = v'(t) = \cos t - t\sin t$

c) $y(t) = \int t\cos t\,dt = t\sin t - \int \sin t\,dt = t\sin t + \cos t + C$

$u = t \quad du = dt$

$v = \sin t \quad dv = \cos t\,dt$

$y(0) = 0 + 1 + C = 3$

$C = 2$

$y(t) = t\sin t + \cos t + 2$

d) $v(t) = t\cos t = 0$ at $t = \frac{\pi}{2}$

$x\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} + 2 = \frac{\pi}{2}(1) + 0 + 2 = \frac{\pi}{2} + 2$

1997 #1 $\quad v(t) = 3t^2 - 2t - 1$

a) $x(t) = \int (3t^2 - 2t - 1)\,dt = t^3 - t^2 - t + C$

$x(2) = 8 - 4 - 2 + C = 5$ so $C = 3$ so $x(t) = t^3 - t^2 - t + 3$

b) avg vel $= \frac{1}{3-0}\int_0^3 v(t)\,dt = \frac{1}{3}\left(x(3) - x(0)\right) = \frac{1}{3}\left(27 - 9 - 3 + 3 - 3\right) = 5$

$3t^2 - 2t - 1 = 5$

$3t^2 - 2t - 6 = 0$ use calc to get $t \approx 1.786$

c) distance $= \int_0^3 |v(t)|\,dt = 17$

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1997 #6 $\quad \frac{dv}{dt} = -2v - 32 = -2(v + 16)$

a) $\int \frac{1}{v+16}\,dv = \int -2\,dt$

$\ln|v + 16| = -2t + c$

$|v + 16| = e^{-2t+c} = Ce^{-2t}$

$v + 16 = \pm Ce^{-2t}$

$v = \pm Ce^{-2t} - 16$

$v(0) = \pm C - 16 = -50$

$\pm C = -34$

$v(t) = -34e^{-2t} - 16$

b) $\lim_{t \to \infty} -34e^{-2t} - 16 = \lim_{t \to \infty} \frac{-34}{e^{2t}} - 16 = 0 - 16 = -16 \ \frac{ft}{sec}$

c) speed $= 20 \Rightarrow$ velocity $= -20$ (she is falling)

$-20 = -34e^{-2t} - 16$

$-4 = -34e^{-2t}$

$\frac{2}{17} = e^{-2t}$

$\ln\left(\frac{2}{17}\right) = -2t$

$t = \frac{\ln\left(\frac{2}{17}\right)}{-2} \approx 1.070$ sec

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2000 #2

a) line passing thru $(0,0)$ and $(3,10)$: slope $= \frac{10}{3}$ so $y = \frac{10}{3}x$

at $t = 2$, $y = \frac{10}{3}(2) = \frac{20}{3}$ so Runner A has velocity of $\frac{20}{3}\ \frac{m}{sec}$

$v(2) = \frac{48}{7}$ so Runner B has velocity of $\frac{48}{7}\ \frac{m}{sec}$

b) for Runner A, $a(t) = $ slope $= \frac{10}{3}\ \frac{m}{sec^2}$

for Runner B, $v'(2) \approx 1.469\ m/sec^2$ (on calculator)

c) Runner A: distance = area under curve $= \frac{1}{2}(3)(10) + 7(10)$

$= 85$ meters

Runner B: $\int_0^{10} v(t)\,dt \approx 83.336$ meters

2001 #3

a) yes, $v(t)$ is increasing at $t=2$ b/c $a(2)>0$

b) at $t=12$ b/c the change in $v(t)$ is found by the integral of $a(t)$. Since $\int_0^6 a(t)dt = -\int_6^{12} a(t)dt$, $\int_0^{12} a(t)dt = 0$ so the velocity would be back at 55 ft/sec.

c) $a(t)=0$ at $t=6,16$

$v(0)=55 \quad v(6)=55+\int_0^6 a(t)dt = 55+2(15)+\frac{1}{2}(4)(15)=115$

$v(16)=55+\int_0^{12} a(t)dt + \int_{12}^{16} a(t)dt = 55+0+-\left(2(15)+\frac{1}{2}(2)(15)\right)=10$

$v(18)=v(16)+\int_{16}^{18} a(t)dt = 10+\frac{1}{2}(2)(15)=25$

max velocity is 115 ft/sec and it occurs at $t=6$ sec

d) it will never be 0 on $[0,18]$ b/c the min velocity (found in part c) is 10 ft/sec

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2002 #3 $\quad v(t)=\sin\left(\frac{\pi}{3}t\right)$

a) $a(4)=v'(4)\approx -.524$

b) Both statements are correct

$v(t)$ is decreasing on $(3,4.5)$ b/c $a(t)<0$ on that interval

speed is increasing on $(3,4.5)$ b/c $a(t)<0$ and $v(t)<0$ on that interval

c) distance $= \int_0^4 |v(t)|dt \approx 2.387$

d) $\int_0^4 v(t)dt = x(t)\Big]_0^4 = x(4)-x(0)$ $\quad$ so $x(4)\approx 3.432$

$1.4323945 = x(4)-2$

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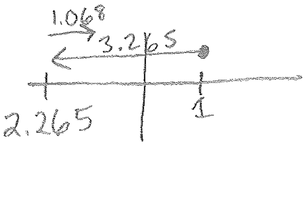
2003 #2 $\quad v(t)=-(t+1)\sin\left(\frac{1}{2}t^2\right)$

a) $a(2)=v'(2)\approx 1.588$, speed is decreasing at $t=2$ b/c $a(2)>0$ and $v(2)<0$

b) $v(t)=0$ at $t\approx 2.507$ b/c $v(t)$ chgs from (−) to (+) at that value

c) distance $= \int_0^3 |v(t)|dt \approx 4.334$

d) $\int_0^{2.507} v(t)dt \approx -3.265483 \quad \int_{2.507}^{3} v(t)dt \approx 1.068$




greatest distance is 2.265 units left of the origin

2004 # 3 $v(t) = 1 - \tan^{-1}(e^t)$

a) $a(2) = v'(2) \approx -.133$

b) speed is increasing at $t=2$ b/c $a(2)<0$ and $v(2)<0$

c) highest point is reached at $t \approx 0.443$ b/c $v(t)>0$ on $(0, .443)$ and $v(t)<0$ on $(.443, \infty)$

d) $\int_0^2 v(t)\,dt = y(t)\Big]_0^2 = y(2) - y(0)$ so $y(2) \approx -1.361$

$-.3606887 = y(2) - 1$

particle is moving away from the origin b/c $y(2)<0$ and $v(2)<0$

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2005 # 5

a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + 12(20) + \frac{1}{2}(8)(20) = 40 + 240 + 80 = 360$ meters

the car traveled a distance of 360 meters from $t=0$ to $t=24$ sec.

b) $v'(4)$ does not exist because there is a sharp turn on the graph of $v(t)$ at $t=4$

$v'(20) = \frac{0-20}{24-16} = \frac{-20}{8} = \frac{-5}{2} \ \frac{m}{sec^2}$

c) $a(t) = \begin{cases} 5 & , 0<t<4 \\ 0 & , 4<t<16 \\ -\frac{5}{2} & , 16<t<24 \end{cases}$

$y = -\frac{5}{2}(x-24)$

$y(20) = -\frac{5}{2}(-4) = 10$

d) $\frac{1}{20-8}\int_8^{20} a(t)\,dt = \frac{1}{12}\left(v(t)\Big]_8^{20}\right) = \frac{1}{12}\left(v(20) - v(8)\right) = \frac{1}{12}(10-20) = \frac{-10}{12} = \frac{-5}{6} \ \frac{m}{sec^2}$

The MVT does not guarantee a value of c on $(8, 20)$ b/c $v'(t) = a(t)$ is not continuous on that interval

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2006 # 4

a) $\frac{1}{80-0}\int_0^{80} a(t)\,dt = \frac{1}{80}\left(v(t)\Big]_0^{80}\right) = \frac{1}{80}\left(v(80) - v(0)\right) = \frac{1}{80}(49-5)$

$= \frac{44}{80} = \frac{11}{20} \ \frac{ft}{sec^2}$

b) $\int_{10}^{70} v(t)\,dt$ is the distance traveled by the rocket, in feet, from $t = 10$ sec to $t = 70$ sec

$\int_{10}^{70} v(t)\,dt \approx 20[22 + 35 + 44] = 20[101] = 2020$ feet

c) $\int_0^{80} a(t)\,dt = \int_0^{80} 3(t+1)^{-\frac{1}{2}}\,dt = 6(t+1)^{\frac{1}{2}}\Big]_0^{80} = 6\sqrt{81} - 6\sqrt{1} = 48$

$\int_0^{80} a(t)\,dt = v(t)\Big]_0^{80} = v(80) - v(0)$

$48 = v(80) - 2$

$v(80) = 50$

$v(80)_{Rocket A} = 49$

$v(80)_{Rocket B} = 50$

so Rocket B is faster