Review of Curve Sketching Answer Key

1969 #7 $\quad y = x + Kx^{-1}$ so $y' = 1 - Kx^{-2} = 1 - \frac{K}{x^2} = \frac{x^2-K}{x^2}$

$(-2)^2 = 0$ or $(-2)^2 - K = 0$

$4 \neq 0 \qquad 4 - K = 0$

$K = 4$

$\boxed{D}$

1969 #17 $\quad y = 5x^4 - x^5, \; y' = 20x^3 - 5x^4, \; y'' = 60x^2 - 20x^3$

$60x^2 - 20x^3 = 0$

$20x^2(3-x) = 0$

$x = 0, 3$

y'': $+$ | $+$ | $-$

x: 0, 3

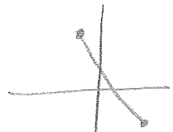
$\boxed{B}$

1969 #21 $\quad f(x) = x^2 + e^{-2x}, \; f'(x) = 2x - 2e^{-2x}$

$f'(0) = 0 - 2e^0 = -2$

$\boxed{B}$

1969 #30



The only choice that must be true is $\boxed{E}$

1973 #22 $\quad f(x) = 3x^5 - 20x^3, \; f'(x) = 15x^4 - 60x^2, \; f''(x) = 60x^3 - 120x$

$60x^3 - 120x = 0$

$60x(x^2 - 2) = 0$

$x = 0, \pm\sqrt{2}$

$f''(x)$: $-$ | $+$ | $-$ | $+$

x: $-\sqrt{2}$, 0, $\sqrt{2}$

$\boxed{B}$

1985 #16 $\quad f(x) = x^3 - 3x^2, \; f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, 2$

$f'(x)$: $+$ | $-$ | $+$

x: $\nearrow$ 0 $\searrow$ 2 $\nearrow$

$\boxed{B}$

1988 #4 $\quad y = -5(x-2)^{-1}, \; y' = 5(x-2)^{-2}, \; y'' = -10(x-2)^{-3} = \frac{-10}{(x-2)^3}$

y'': $+$ | $-$

x: 2

$\boxed{E}$

1993 #15 $\quad f(x) = (x-2)(x-3)^2, \; f'(x) = (x-3)^2 + (x-2)(2)(x-3)$

$= (x-3)[x-3+2(x-2)] = (x-3)(3x-7) = 0$

$x = 3, \frac{7}{3}$

$f'(x)$: $+$ | $-$ | $+$

x: $\nearrow$ $\frac{7}{3}$ $\searrow$ 3 $\nearrow$

$\boxed{D}$

1993 #21 $\quad y = x^{-2} - x^{-3}, \; y' = -2x^{-3} + 3x^{-4}, \; y'' = 6x^{-4} - 12x^{-5}$

$= \frac{6}{x^4} - \frac{12}{x^5} = \frac{6x-12}{x^5}$

y'': $+$ | $-$ | $+$

0: 0, 2

$6x - 12 = 0 \qquad x^5 = 0$

$x = 2 \qquad x = 0$

($f(x)$ is undefined at $x = 0$) $\longrightarrow$ $\boxed{C}$

1997 #5

$y = 3x^4 - 16x^3 + 24x^2 + 48$

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48 = 0$

$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$

y'': $+$ | $-$ | $+$

x: $\frac{2}{3}$, 2

$\boxed{E}$

1997 #85

$f'(x) = e^x - 3x^2 = 0$

$x = 3.733, .910, -.459$

$f'(x)$: $-$ | $+$ | $-$ | $+$

x: $-.459$, $.910$, 3.733

$\searrow$ $\nearrow$ $\searrow$ $\nearrow$

$\boxed{C}$

1998 #1 $y = \frac{1}{3}x^3 + 5x^2 + 24$, $y' = x^2 + 10x$, $y'' = 2x + 10 = 0$

$x = -5$

$\boxed{D}$

1998 #19 $f''(x) = x(x+1)(x-2)^2 = 0$

$x = 0, -1, 2$

$f''(x)$: $+$ | $-$ | $+$ | $+$

x: -1, 0, 2

$\boxed{C}$

1998 #79 rel max occurs where the derivative changes from positive to negative $\boxed{A}$

1998 #89 $f'(x) = (x^2 - 4)\underbrace{g(x)}_{<0} = (x+2)(x-2)\underbrace{g(x)}_{<0}$

$f'(x)$: $-$ | $+$ | $-$

x: -2, 2

$\searrow$ $\nearrow$ $\searrow$

$\boxed{B}$

1985 #6

a) $f'(x)$: $+$ | $-$ | $+$ | $+$

x: -3 $\nearrow$ -2 $\searrow$ 0 $\nearrow$ 2 $\nearrow$ 3

a rel. max occurs at $x = -2$ b/c $f'(x)$ chgs from $(+)$ to $(-)$

a rel. min occurs at $x = 0$ b/c $f'(x)$ chgs from $(-)$ to $(+)$

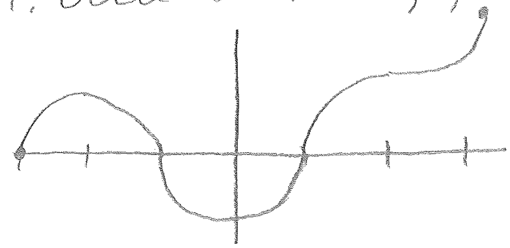
b) $f''(x)$: $-$ | $+$ | $-$ | $+$

x: -3, -1, 1, 2, 3

$f(x)$ is concave up on $(-1, 1) \cup (2, 3)$ b/c $f''(x) > 0$

c) p.o.i. occur at $x = -1, 1,$ and 2 b/c $f''(x)$ changes sign

d)



$f(x)$: $\nearrow$ $\searrow$ $\searrow$ $\nearrow$ $\nearrow$ $\nearrow$

x: -3, -2, -1, 0, 1, 2, 3

1992 #1 $\quad f(x) = 3x^5 - 5x^3 + 2$

a) $f'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1) = 0$

$f'(x)$	+		−		−		+
x		-1		0		1	

f is inc. on $(-\infty,-1) \cup (1,\infty)$
b/c $f'(x) > 0$

b) $f''(x) = 60x^3 - 30x = 30x(2x^2-1) = 0$

$f''(x)$	−		+		−		+
x		$-\sqrt{\frac{1}{2}}$		0		$\sqrt{\frac{1}{2}}$	

$x = 0, \pm\sqrt{\frac{1}{2}}$

f is concave up on $(-\sqrt{\frac{1}{2}}, 0) \cup (\sqrt{\frac{1}{2}}, \infty)$
b/c $f''(x) > 0$

c) $f'(x) = 0$ at $x = 0, \pm 1$
$f(0) = 2$, $f(1) = 0$ $f(-1) = 4$ so $y = 2$, $y = 0$ and $y = 4$

1991 #5

a) Critical #s: $0, 1, 2, -1, -2$
endpoints: $-3, 3$

$f(0) = 1$ $\quad f(-1) = 0$
$f(1) = 0$ $\quad f(-2) = -1$
$f(2) = -1$ $\quad f(3) < 0$ but > -1
same for $f(-3)$

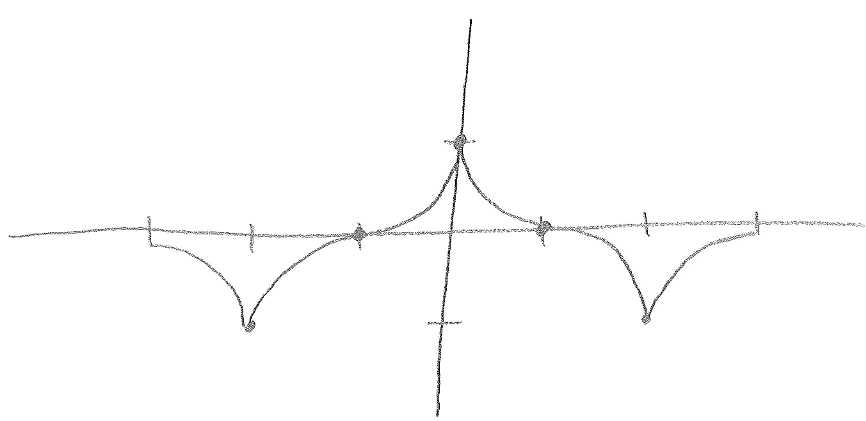
abs max at $x = 0$, abs min at $x = -2, 2$

b)

$f''(x)$		−		−		+		+		−		−	
x	-3		-2		-1		0		1		2		3

p.o.i. occur at $x = -1$ and $x = 1$ b/c $f''(x)$ chgs sign

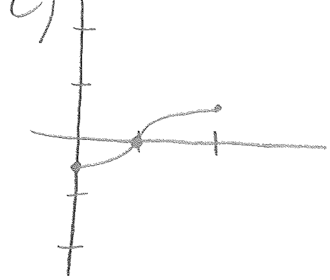
c)



1993 #5

a) $f'(x) = \begin{cases} x, & 0 < x < 1 \\ -x+2, & 1 \le x < 2 \end{cases}$

b) $f(x) = \begin{cases} \frac{1}{2}x^2 + C, & 0 < x < 1 \\ -\frac{1}{2}x^2 + 2x + K, & 1 < x < 2 \end{cases}$

c) 

$\lim_{x \to 1^-} f(x) = \frac{1}{2} + C = 0$ so $C = -\frac{1}{2}$

$\lim_{x \to 1^+} f(x) = -\frac{1}{2} + 2 + K = 0$ so $K = -\frac{3}{2}$

so $f(x) = \begin{cases} \frac{1}{2}x^2 - \frac{1}{2}, & 0 < x < 1 \\ -\frac{1}{2}x^2 + 2x - \frac{3}{2}, & 1 < x < 2 \end{cases}$

1996 #1

a)

$f'(x)$		+		−		−		+	
x	-3	↗	-2	↘	1	↘	4	↗	5

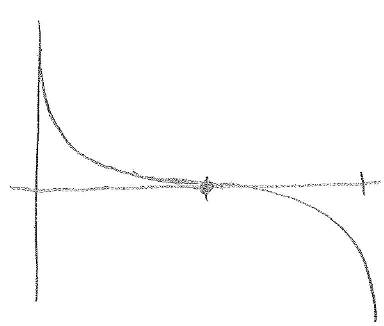
rel max is at $x = -2$ b/c $f'(x)$ changes from (+) to (−)

b) rel min is at $x = 4$ b/c $f'(x)$ changes from (−) to (+)

c)

$f''(x)$		−		+		−		+	
x	-3	∩	-1	∪	1	∩	3	∪	5

concave up on $(-1, 1) \cup (3, 5)$ b/c $f'(x)$ is increasing

d) 

$f(x)$		∪ ↘		∩ ↘	
x	0		1		2

2000 #3

a)

$f'(x)$		+		−		+		+	
x	-7		-5		-1		5		7

rel. min. at $x=-1$ b/c $f'(x)$ chgs from $(-)$ to $(+)$

b) rel. max. at $x=-5$ b/c $f'(x)$ chgs from $(+)$ to $(-)$

c)

$f''(x)$		−		+		−		−		+	
x	-7		-3		2		3		5		7

$f''(x)<0$ on $(-7,-3)\cup(2,3)\cup(3,5)$

d) will either be at $x=-5$ (rel max) or
$x=-7, 7$ (endpoints)
it can't be at $x=-7$ b/c the function increases from that point
so it's either at $x=-5$ or 7
if we choose a low point between them like $x=-1$, we know that

$\int_{-5}^{-1} f'(x)\,dx = f(-1)-f(-5) < 0$ so $f(-5) > f(-1)$

$\int_{-1}^{7} f'(x)\,dx = f(7)-f(-1) > 0$ so $f(7) > f(-1)$

but the distance between $f(-1)$ and $f(7)$ is greater than the distance between $f(-1)$ and $f(5)$ because there is "more graph" above the x-axis from $x=-1$ to 7 than below the graph from $x=-5$ to -1. So the abs max is at $x=7$.

2002 #4

a) $g(-1)=\int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -\frac{1}{2}(1)(3) = -\frac{3}{2}$ $\quad g'(-1)=f(-1)=0$ $\quad g''(-1)=f'(-1)=3$

b)

$g'(x)$		−		+		−	
x	-2	↘	-1	↗	1	↘	2

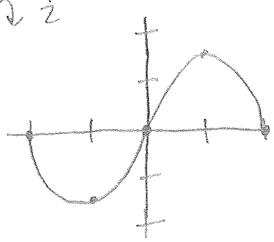
$g(x)$ is inc. on $(-1,1)$ b/c $g'(x)=f(x)>0$

c)

$g''(x)$		+		−	
x	-2		0		2

$g(x)$ is concave down on $(0,2)$ b/c $g''(x)=f'(x)<0$ ($f(x)$ is decreasing)

d)



$g(0)=\int_0^0 f(t)\,dt=0$

$g(1)=\int_0^1 f(t)\,dt=\frac{3}{2}$ $\quad g(2)=\int_0^2 f(t)\,dt=0$

$g(-2)=-\int_{-2}^{0} f(t)\,dt=0$

2003 #4

a) $f'(x)$: $+$ on $(-3,-2)$, $-$ on $(-2,2)$, $-$ on $(2,4)$; x: -3, -2, 2, 4

f is inc. on $(-3,-2)$ b/c $f'>0$

b) $f''(x)$: $-$ on $(-3,0)$, $+$ on $(0,2)$, $-$ on $(2,4)$; x: -3, 0, 2, 4

poi occur at $x=0,2$ b/c $f''(x)$ chgs sign ($f'(x)$ chgs direction)

c) $f'(0)=-2$ $\quad y-3=-2x$

d) $\int_{-3}^{0} f'(x)\,dx = f(x)\Big]_{-3}^{0} = f(0)-f(-3)$ $\qquad \int_{0}^{4} f'(x)\,dx = f(x)\Big]_{0}^{4} = f(4)-f(0)$

$\frac{3}{2} = 3 - f(-3)$ $\qquad -\left(8-\frac{1}{2}(4\pi)\right) = f(4)-3$

$f(-3)=\frac{3}{2}$ $\qquad f(4) = -5+2\pi$

2004 #5

a) $g(0)=\int_{-3}^{0} f(t)\,dt = 3+\frac{1}{2}(3)(1)=\frac{9}{2}$, $g'(0)=f(0)=1$

b) $g'(x)$: $-$ on $(-5,-4)$, $+$ on $(-4,1)$, $+$ on $(1,3)$, $-$ on $(3,4)$; x: $-5 \searrow -4 \nearrow 1 \nearrow 3 \searrow 4$

rel max at $x=3$ b/c $g'(x)=f(x)$ chgs from $(+)$ to $(-)$

c) abs min is either at $x=-4$ or 4

$g(4)=\int_{-3}^{4} f(t)\,dt = \frac{9}{2}+2-\frac{1}{2}\pi = \frac{13-\pi}{2}$ $\qquad g(-4) = -\int_{-4}^{-3} f(t)\,dt = -\frac{1}{2}(2) = -1$

abs min is at $x=-4$

d) $g''(x)$: $+$ on $(-5,-3)$, $-$ on $(-3,0)$, $-$ on $(0,1)$, $+$ on $(1,2)$, $-$ on $(2,4)$; x: -5, -3, 0, 1, 2, 4

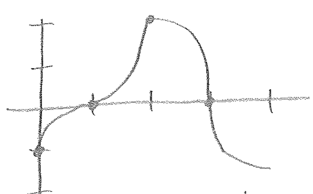
poi occur at $x=-3,1,2$ b/c $g''(x)=f'(x)$ chgs sign

2005 #4

a) $f'(x)$: $+$ on $(0,1)$, $+$ on $(1,2)$, $-$ on $(2,3)$, $-$ on $(3,4)$; x: $0 \nearrow 1 \nearrow 2 \searrow 3 \searrow 4$

rel max at $x=2$ b/c $f'(x)$ chgs from $(+)$ to $(-)$

b)



c) $g'(x)=f(x)$ $\quad g'(x)$: $-$ on $(0,1)$, $+$ on $(1,2)$, $+$ on $(2,3)$, $-$ on $(3,4)$; x: $0 \searrow 1 \nearrow 2 \nearrow 3 \searrow 4$

rel max at $x=3$ b/c $g'(x)=f(x)$ chgs from $(+)$ to $(-)$

rel min at $x=1$ b/c $g'(x)=f(x)$ chgs from $(-)$ to $(+)$

d) $g''(x)=f'(x)$ $\quad$ g has a poi at $x=2$ b/c $g''(x)=f'(x)$ chgs sign