Review of Antiderivative Word Probs - Answers

2002 #2

a) $\int_9^{17} E(t)\,dt \approx 6004$ people

b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 90064.0548599 + 13984.110$

$\approx \$104048$

c) $H'(t) = E(t) - L(t)$ so $H'(17) = E(17) - L(17) \approx 380.4878$

$380.4878 - 760.76923 = -380.281$

$H(17)$ means that there are 3725 people in the park at 5:00 pm.

$H'(17)$ means that at 5:00 pm people are leaving the park at a rate of 380.281 people per hour.

d) $H'(t) = E(t) - L(t) = 0$ at $t \approx 15.794815$

$H(9) = 0$ $\quad H(15.794815) \approx 3950.680$ $\quad H(23) = 0$

max is at $t \approx 15.794815$

2004 #1 $\quad F(t) = 82 + 4\sin(\frac{1}{2}t)$, $0 \le t \le 30$

a) $\int_0^{30} F(t)\,dt \approx 2474$ cars

b) Traffic flow is decreasing at $t=7$ b/c $F'(7) < 0$

c) $\frac{1}{15-10}\int_{10}^{15} F(t)\,dt \approx \frac{1}{5}(409.49621) \approx 81.899 \frac{\text{cars}}{\text{min}}$

d) $\frac{1}{15-10}\int_{10}^{15} F'(t)\,dt = \frac{1}{5}\left(F(t)\Big]_{10}^{15}\right) = \frac{1}{5}\left(F(15) - F(10)\right)$

$= \frac{1}{5}\left(85.752 - 78.164303\right) = 1.518 \frac{\text{cars}}{\text{min}^2}$

2005 #2 $\quad R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right) \quad S(t) = \frac{15t}{1+3t}$

a) $\int_0^6 R(t)\,dt \approx 31.816$ cubic yards

b) $Y(t) = 2500 + \int_0^t [S(x) - R(x)]\,dx$

c) $Y'(t) = S(t) - R(t)$ so $Y'(4) = S(4) - R(4) = -1.909$ cubic yards/hour

d) $Y'(t) = S(t) - R(t) = 0$ at $t \approx 5.1178653$

$Y(0) = 2500$

$Y(6) = 2500 + \int_0^6 [S(x) - R(x)]\,dx \approx 2493.277$

$Y(5.1178653) = 2500 - 7.630517 \approx 2492.369$

So the amount of sand is a minimum at $t \approx 5.118$ and that minimum value is 2492.369 cubic yards.

2006 #2 $\quad L(t) = 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$

a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

b) $60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right) = 150$ at $t \approx 12.42831,\ 16.121657$

$L(t) \geq 150$ on $[12.42831, 16.121657]$

$\frac{1}{16.121657 - 12.42831} \int_{12.42831}^{16.121657} L(t)\,dt \approx \frac{1}{3.693347}(736.54986) \approx 199.426$ cars/hour

c) $500 \int_a^b L(t)\,dt > \cancel{200{,}000}$

so $\int_a^b L(t)\,dt > 400$

using an estimated guess at the bounds, I found $\int_{14}^{16} L(t)\,dt \approx 412.266 > 400$

$\therefore$ The intersection requires a traffic signal.

2007 #2

a) $\int_0^7 100t^2 \sin\sqrt{t}\,dt \approx 8264$ gallons

b) water is decreasing when $g(t) > f(t)$ or $g(t) - f(t) > 0$. This occurs on $[0, 1.617) \cup (3, 5.076)$.

c) Amount of water at time $t = a$ is $\int_0^a [f(t) - g(t)]\,dt + 5000$. So the derivative of this is $f(a) - g(a)$ and the critical values are $t = 1.617, 3, 5.076$.

Test (let $w(t)$ = the amt of water in the tank)

$w(0) = 5000$

$w(1.617) = \int_0^{1.617} [f(t) - 250]\,dt + 5000 \approx 4719.223$

$w(3) = \int_0^3 [f(t) - 250]\,dt + 5000 \approx 5126.591$

$w(5.076) = w(3) + \int_3^{5.076} (f(t) - 2000)\,dt + 5000 \approx 3890.29$

$w(7) = w(3) + \int_3^7 (f(t) - 2000)\,dt + 5000 \approx 4387.216$

Water is greatest at $t = 3$ with about 5127 gallons.

2009 #2 a) $\int_0^2 R(t)\,dt = 980$ people

b) $R'(t) = 2760t - 2025t^2 = 0$ at $t = \frac{184}{135} \approx 1.363$

$R(0) = 0 \quad R\left(\frac{184}{135}\right) \approx 854.527 \quad R(2) = 120$

The rate is a maximum at $t = \frac{184}{135}$ or 1.363

c) $\int_1^2 w'(t)\,dt = w(2) - w(1) = 387.5$

d) $\frac{1}{980} \int_0^2 w'(t)\,dt = \frac{760}{980} \approx .776$ hours