A.P. Calculus AB **Trig Quiz #3 REVIEW** Name KEY

NO CALCULATORS! All answers must be justified!

1. Determine whether or not the MVT applies to the function $f(x)=\sin x-x$ on $[-2\pi,-\pi]$. If so, find the value(s) of c as defined in the theorem.

$f(x)$ is continuous on $[-2\pi,\pi]$ ✓, $f'(x)=\cos x-1$ ∴ $f(x)$ is differentiable ✓ ∴ MVT applies

Slope between endpoints $= \dfrac{f(-\pi)-f(-2\pi)}{-\pi-(-2\pi)} = \dfrac{(0-(-2\pi))-(0-(-\pi))}{-\pi+2\pi} = \dfrac{\pi-2\pi}{-\pi+2\pi} = \dfrac{-\pi}{\pi} = -1$

$f'(c) = \cos c - 1 = -1$

for $\cos c = 0$ ∴ $\boxed{c=-\frac{3\pi}{2}}$ on $[-2\pi,\pi]$

2. Determine whether or not Rolle's theorem applies to the function $f(x)=\sec x$ on $\left[-\frac{7\pi}{6},-\frac{5\pi}{6}\right]$

If so, find the value(s) of c as defined in the theorem.

$f(x)$ is continuous on $\left[-\frac{7\pi}{6},-\frac{5\pi}{6}\right]$ ✓; $f'(x)=\sec x\tan x$ ✓ ∴ differentiable; $\sec -\frac{7\pi}{6} = -\frac{2\sqrt{3}}{3} = \sec -\frac{5\pi}{6}$ ✓ ∴ Rolle's Thm Applies

∴ $f'(c) = \sec c \tan c = 0$

for $\sec c = 0$ No solution ~~∉~~

or $\tan c = 0$ $\boxed{c=-\pi}$ on $\left[-\frac{7\pi}{6},-\frac{5\pi}{6}\right]$

3. Determine whether or not the MVT applies to the function $f(x)=5x+\cot\left(\frac{x}{2}\right)$ on $\left[\frac{\pi}{3},\frac{5\pi}{3}\right]$.

If so, find the value(s) of c as defined in the theorem, (leave answer(s) as inverse trig functions.)

$f(x)$ is continuous ✓; $f'(x) = 5-\frac{1}{2}\csc^2\left(\frac{x}{2}\right)$ ✓ ∴ $f(x)$ is differentiable ∴ MVT applies

Slope between endpoints $= \dfrac{f\left(\frac{5\pi}{3}\right)-f\left(\frac{\pi}{3}\right)}{\frac{5\pi}{3}-\frac{\pi}{3}} = \dfrac{\left(5\left(\frac{5\pi}{3}\right)+\cot\left(\frac{5\pi}{6}\right)\right)-\left(5\left(\frac{\pi}{3}\right)+\cot\frac{\pi}{6}\right)}{\frac{5\pi}{3}-\frac{\pi}{3}} = \dfrac{\left(\frac{25\pi}{3}-\sqrt{3}\right)-\left(\frac{5\pi}{3}+\sqrt{3}\right)}{\frac{4\pi}{3}}$

$= \dfrac{\frac{20\pi}{3}-2\sqrt{3}}{\frac{4\pi}{3}} = \boxed{5-\frac{6\sqrt{3}}{4\pi}}$

$f'(c) = 5-\frac{1}{2}\csc^2\left(\frac{c}{2}\right) = 5-\frac{6\sqrt{3}}{4\pi}$

∴ $\csc^2\frac{c}{2} = \frac{6\sqrt{3}}{4\pi}$ → $\sin^2\left(\frac{c}{2}\right) = \frac{4\pi}{6\sqrt{3}}$

$\sin\left(\frac{c}{2}\right) = \pm\sqrt{\frac{4\pi}{6\sqrt{3}}}$ $\frac{c}{2} = \sin^{-1}\left(\pm\sqrt{\frac{4\pi}{6\sqrt{3}}}\right)$.

$\boxed{c = 2\sin\sqrt{\frac{4\pi}{6\sqrt{3}}} \text{ and } 2\sin-\sqrt{\frac{4\pi}{6\sqrt{3}}}}$

4. Given the function $y = 2\sin x + 2\cos x$ on the interval $(-\pi, \pi)$, find:

a) the intervals of direction: increasing: $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$ b/c $\frac{dy}{dx} > 0$

decreasing: $\left(-\pi, -\frac{3\pi}{4}\right) \cup \left(\frac{\pi}{4}, \pi\right)$ b/c $\frac{dy}{dx} < 0$

$\frac{dy}{dx} = 2\cos x - 2\sin x$

$f'(x) = 0 \quad 2\cos x = 2\sin x$

$\cos x = \sin x$

$1 = \tan x$

$x = -\frac{3\pi}{4}, \frac{\pi}{4}$

$f'(x)$: $\ominus$ ↘ $-\frac{3\pi}{4}$ $\oplus$ ↗ $\frac{\pi}{4}$ $\ominus$ ↘

b) the relative extrema min: $\left(-\frac{3\pi}{4}, -2\sqrt{2}\right)$ b/c $\frac{dy}{dx}$ changes from $\ominus$ to $\oplus$ at $x = -\frac{3\pi}{4}$

max: $\left(\frac{\pi}{4}, 2\sqrt{2}\right)$ b/c $\frac{dy}{dx}$ changes from $\oplus$ to $\ominus$ at $x = \frac{\pi}{4}$

c) the point(s) of inflection: $\left(-\frac{\pi}{4}, 0\right)$ and $\left(\frac{3\pi}{4}, 0\right)$ because $\frac{dy}{dx}$ changes signs at $x = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\frac{d^2y}{dx^2} = -2\sin x - 2\cos x$

P.o.I. when $\frac{d^2y}{dx^2} = 0$: $0 = -2\sin x - 2\cos x$

$2\sin x = -2\cos x$

$\tan x = -1$

$\therefore x = -\frac{\pi}{4}, \frac{3\pi}{4}$

$\oplus$ $-\frac{\pi}{4}$ $\ominus$ $\frac{3\pi}{4}$ $\oplus$

d) concavity intervals up: $\left(-\pi, -\frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$ b/c $\frac{d^2y}{dx^2} > 0$

down: $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right.$ b/c $\frac{d^2y}{dx^2} < 0$

e) Sketch the graph based on your answers to parts a – d.