A.P. Calculus AB **4.3 – 4.5 Quiz Review** Solutions

NO CALCULATORS!

Avg. value theorem: $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

1. Given $f(x) = x^2 - 2x + 3$, find a) average value in the interval [0, 3] b) find the value of c guaranteed by the theorem

$f(c) = \frac{1}{3-0}\int_0^3 x^2 - 2x + 3\,dx$

$= \frac{1}{3}\cdot\left[\frac{x^3}{3} - \frac{2x^2}{2} + 3x\right]_0^3$

$\frac{1}{3}\cdot\frac{27}{3} - 3^2 + 9$

$= \frac{1}{3}(9 - 9 + 9)$

$= 3$

$f(c) = 3$

Avg. value = 3

b) $x^2 - 2x + 3 = 3$

$x^2 - 2x = 0$

$x(x-2) = 0$ $x = 0, x = 2$

$\boxed{c = 2}$, $\boxed{c = 0}$

2. Given $f(x) = \sec^2 x$, find average value in the interval $[-\pi/4, \pi/4]$

$f(c) = \frac{1}{\pi/4 - -\pi/4}\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = \frac{1}{\pi/2}\cdot \tan x\Big]_{-\pi/4}^{\pi/4} = \frac{2}{\pi}\cdot\left[\tan\pi/4 - \tan(-\pi/4)\right] = \frac{2}{\pi}[1 - (-1)]$

$f(c) = \frac{2}{\pi}\cdot 2 = \boxed{\frac{4}{\pi}}$

SFTC: $\frac{d}{dx}\int_a^{p(x)} f(t)\,dt = f(p(x))\cdot p'(x)$

3. If $f(x) = \int_{-2}^{-2x^2} \frac{t}{4-t^3}\,dt$, find $\frac{d}{dx}f(x)$. use $\boxed{\text{SFTC}}$

$= \frac{-2x^2}{4-(-2x^2)^3}\cdot -4x = \frac{8x^3}{4 + 8x^6}$

$= \boxed{\frac{2x^3}{1+2x^6}}$

4. If $f(x) = \int_{-x}^{3\sqrt{x}} 1 - 2t\,dt$, find $\frac{d}{dx}f(x)$.

$\int_{-x}^{3\sqrt{x}} 1 - 2t\,dt = t - \frac{2t^2}{2}\Big]_{-x}^{3\sqrt{x}} = 3\sqrt{x} - 9x - (-x - x^2)$

$= 3\sqrt{x} - 9x + x + x^2$

$= 3x^{1/2} - 8x + x^2$

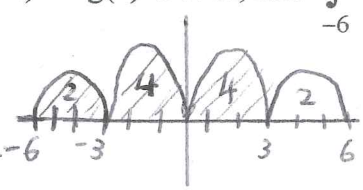
$\frac{d}{dx}(3x^{1/2} - 8x + x^2) =$

$3\cdot\frac{1}{2}x^{-1/2} - 8 + 2x$

$= \boxed{\frac{3}{2\sqrt{x}} - 8 + 2x}$

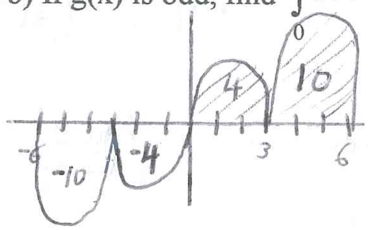
5. Let $\int_{-3}^{6} g(x)\,dx = 10$ and $\int_{3}^{0} g(x)\,dx = -4$ $= \int_0^3 g(x)\,dx = 4$

a) If g(x) is even, find $\int_{-6}^{3} g(x)\,dx$



$\int_{-6}^{3} g(x)\,dx = \boxed{10}$

b) If g(x) is odd, find $\int_{0}^{6} g(x)\,dx$



$\int_0^6 g(x)\,dx = \boxed{14}$

4.3-4.5 Quiz Review (continued)

6. If $\int_3^7 f(x)\,dx = -4$

a) $\int_7^3 2f(x)\,dx = 2\left[-\int_3^7 f(x)\,dx\right]$

$2\cdot(-(-4)) = \boxed{8}$

b) $\int_7^3 [3f(x)-2]\,dx = 3\int_7^3 f(x)\,dx - \int_7^3 2\,dx$

$3\cdot(4) = 12$

$2x\Big]_7^3 = 6-14 = -8$

$= 12-(-8) = \boxed{20}$

7. Evaluate $\int \frac{2}{x^2}\sec\left(\frac{3}{x}\right)\tan\left(\frac{3}{x}\right)dx$

$u = \frac{3}{x} = 3x^{-1}$

$\frac{du}{dx} = -3x^{-2}$

$\frac{du}{dx} = \frac{-3}{x^2}$

$dx = -\frac{x^2}{3}du$

$\int \frac{2}{\cancel{x^2}}\sec(u)\tan(u)\cdot -\frac{\cancel{x^2}}{3}du$

$= -\frac{2}{3}\int \sec u\tan u\,du$

$= -\frac{2}{3}\sec u + C$

$= \boxed{-\frac{2}{3}\sec\left(\frac{3}{x}\right)+C}$

8. Evaluate $\int 5x\sqrt{2-x}\,dx = \int 5x(2-x)^{1/2}dx$

$u = 2-x$

$\frac{du}{dx} = -1$

$dx = -du$

$x = 2-u$

$\int 5x\cdot u^{1/2}(-du)$

$\int 5(2-u)u^{1/2}\cdot(-du)$

$= \int -10u^{1/2}+5u^{3/2}du$

$= \frac{-10u^{3/2}}{3/2} + \frac{5u^{5/2}}{5/2} + C$

$= \frac{2}{3}\left(-10u^{3/2}\right) + \frac{2}{5}\left(5u^{5/2}\right) + C$

$= -\frac{20}{3}u^{3/2} + 2u^{5/2} + C$

$= \boxed{-\frac{20}{3}(2-x)^{3/2} + 2(2-x)^{5/2} + C}$

9. Evaluate $\int_4^9 \frac{x+1}{\sqrt{x}}dx = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}$

$\int_4^9 x^{1/2} + x^{-1/2}dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$

$\frac{2}{3}x^{3/2} + 2x^{1/2}\Big]_4^9$

$= \frac{2}{3}(9)^{3/2} + 2(9)^{1/2} - \left(\frac{2}{3}(4)^{3/2} + 2(4)^{1/2}\right)$

$\frac{2}{3}(27) + 2(3) - \frac{2}{3}(8) - 2(2)$

$18 + 6 - \frac{16}{3} - 4 = \boxed{\frac{44}{3}}$

10. Evaluate $\int_0^{\frac{\pi}{3}} \tan^2 x\sec^2 x\,dx$

if $x=0$, $u=\tan 0 = 0$
if $x=\pi/3$, $u=\tan(\pi/3)=\sqrt{3}$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int_0^{\pi/3} u^2\cdot\cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}}$

$\frac{u^3}{3}\Big]_0^{\sqrt{3}} = \frac{1}{3}(\sqrt{3})^3 - \frac{1}{3}(0)^3 = \frac{1}{3}(3\sqrt{3}) = \boxed{\sqrt{3}}$