AP Calculus AB 4-2, 4-6 Quiz Review **Name** Solution Key

Calculators permitted.

1. Find the sum: $\sum_{i=2}^{4}\left[(i+1)^2-(2-i)^3\right]$

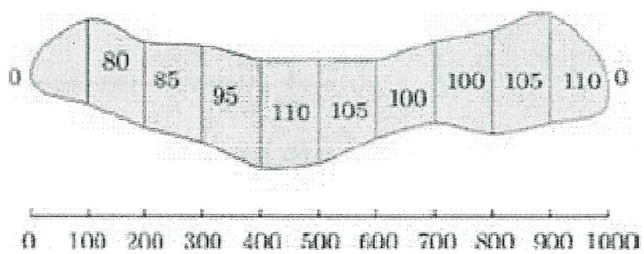
$(2+1)^2-(2-2)^3+(3+1)^2-(2-3)^3+(4+1)^2-(2-4)^3$

$9 - 0 + 16 - (-1) + 25 - (-8) = \boxed{59}$

2. Use Sigma notation to write the sum: $\frac{2}{\sqrt[3]{5-2}}+\frac{4}{\sqrt[3]{5-4}}+\frac{6}{\sqrt[3]{5-6}}+\frac{8}{\sqrt[3]{5-8}}$

$\sum_{i=1}^{4} \frac{2i}{\sqrt[3]{5-2i}}$

3. The width, in feet, at various points along the fairway of a hole on a golf course is given to the right. If one pound of fertilizer covers 200 square feet, estimate the amount of fertilizer needed to fertilize the fairway using trapezoids.



$A = \frac{w}{2}\left[h_1 + 2h_2 + 2h_3 + 2h_3 + \ldots + h_n\right]$

$A \approx \frac{100}{2}\left[0+2(80)+2(85)+2(95)+2(110)+2(105)+2(100)+2(100)+2(105)+2(110)+0\right]$

$A \approx 50(1780) = 89000 \text{ ft}^2$

fertilizer needed $= 89000 \text{ ft}^2 \cdot \frac{1 \text{ lb fertilizer}}{200 \text{ ft}^2} =$ $\boxed{\text{445 pounds of fertilizers}}$

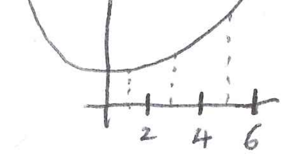
4. Use 3 middle rectangles to approximate the area of the region bounded by $f(x) = x^2 + 3$, the x-axis, $x = 0$, and $x = 6$.

width $= \frac{b-a}{n} = \frac{6-0}{3} = 2$

$A = 2f(1) + 2f(3) + 2f(5)$

$A = 2(4) + 2(12) + 2(28)$

$= \boxed{88}$



$f(1) = 1^2 + 3 = 4$

$f(3) = 3^2 + 3 = 12$

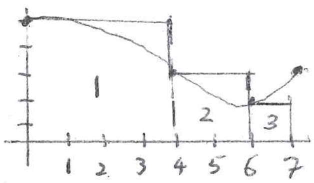
$f(5) = 5^2 + 3 = 28$

5. Use the table of values on the right to estimate the below:

x	0	4	6	7	10
$f(x)$	5	3	2	3	5

a. Use 3 left-handed rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 7]

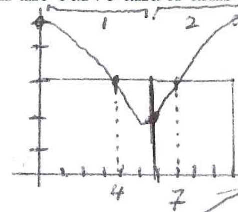
x	0	4	6	7
f(x)	5	3	2	3



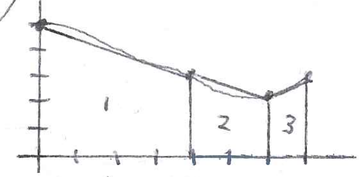
$$A = 4\cdot f(0) + 2\cdot f(4) + 1\cdot f(6)$$
$$= 4(5) + 2(3) + 1(2)$$
$$= \boxed{28}$$

b. Use 2 middle rectangles with intervals indicated by the table to estimate the area between the curve and x-axis on [0, 10]

x	0	4	6	7	10
f(x)	5	3	2	3	5

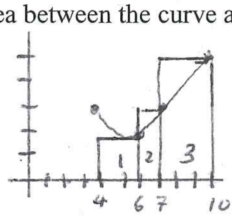


$$A = 6\cdot f(4) + 4\cdot f(7)$$
$$= 6(3) + 4(3)$$
$$= \boxed{30}$$



c. Use 3 right-handed rectangles with intervals indicated by the table to estimate area between the curve and x-axis on [4, 10]

x	4	6	7	10
f(x)	3	2	3	5



$$A = 2f(6) + 1f(7) + 3\cdot f(10)$$
$$= 2(2) + 1(3) + 3(5)$$
$$= 4 + 3 + 15 = \boxed{22}$$

d. Use 3 trapezoids with interval indicated by the table to estimate area between the curve and x-axis on [0, 7]

x	0	4	6	7
f(x)	5	3	2	3

$$A = \frac{w}{2}[h_1 + h_2]$$

$$A = \frac{4}{2}[f(0)+f(4)] + \frac{2}{2}[f(4)+f(6)] + \frac{1}{2}[f(6)+f(7)]$$
$$= 2(5+3) + 1(3+2) + \tfrac{1}{2}(2+3)$$
$$= 2(8) + 1(5) + \tfrac{1}{2}(5)$$
$$= 10 + 5 + 5/2 = \boxed{17.5 \text{ or } 35/2}$$

6. Given the region bounded by $g(x) = 6 - x^2$, the x-axis, $x = -1$, and $x = 2$. Use the limit definition to find the exact area of the region.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \text{width} \cdot f(\text{left} + \text{width}\cdot i)$$

$$\text{width} = \frac{b-a}{n} = \frac{2--1}{n} = \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\cdot f\left(-1 + \frac{3}{n}i\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\cdot \left[6 - \left(-1 + \frac{3i}{n}\right)^2\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[6 - \left(-1+\frac{3i}{n}\right)\left(-1+\frac{3i}{n}\right)\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[6 - \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[5 + \frac{6i}{n} - \frac{9i^2}{n^2}\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{15}{n} + \frac{18}{n^2}i - \frac{27i^2}{n^3}\right]$$

$$= \lim_{n\to\infty} \left[\frac{15}{n}\sum_{i=1}^{n} 1 + \frac{18}{n^2}\sum_{i=1}^{n} i - \frac{27}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{15}{n}(n) + \frac{18}{n^2}\left(\frac{n^2}{2} + \frac{n}{2}\right) - \frac{27}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{15n}{n} + \frac{18n^2}{2n^2} + \cancel{\frac{18n}{2n^2}} - \frac{27n^3}{3n^3} - \cancel{\frac{27n^2}{2n^3}} - \cancel{\frac{27n}{6n^3}}\right]$$

$$= 15 + \frac{18}{2} - \frac{27}{3} = 15 + 9 - 9 = \boxed{15}$$