AP Calculus Notes 10.2 – Plane Curves & Parametric Equations **Name** ___________________

Definition: **Parametric equations** in the plane are a pair of functions

$$x = f(t) \quad \text{and} \quad y = g(t)$$

which describe the x and y coordinates of some curve in the plane.

Sketching a Plane Curve

A set of **parametric equations** sketches out a **plane curve** that represents the path that a point P takes as it wanders about the xy-plane. As P makes its way about the plane, its x- and y- coordinates can be described by the functions

$$x = f(t) \quad \text{and} \quad y = g(t)$$

where f and g are functions of time, t. In this situation, t is called the **parameter.** Each set of coordinates (x, y) is determined from a value chosen for the parameter, t. By plotting the resulting points in the order of *increasing* values of t, you trace the curve in a specific **direction**, also known as the **orientation** of the curve. The **orientation** is indicated by arrows on the curve.

Example 1:

Construct a table and sketch the curve represented by this set of parametric equations:

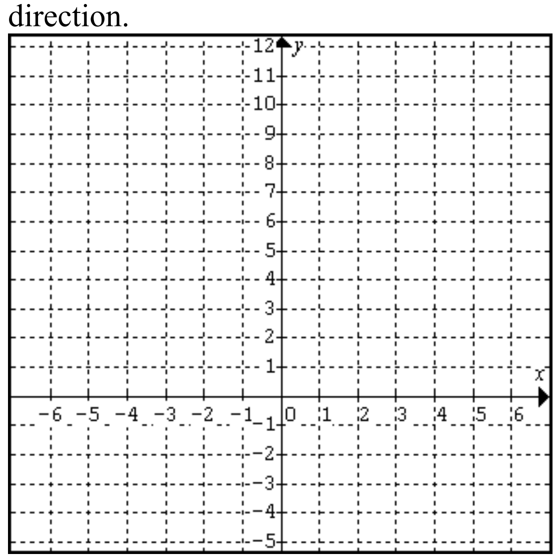
$x = t^2$ $\quad -2 \le t \le 2$

$y = t^2 - 4t$

Construct a table with t, x, and y.

t	***x***	***y***
-2		
-1		
0		
1		
2		

Graph the x-y coordinates and show the direction.



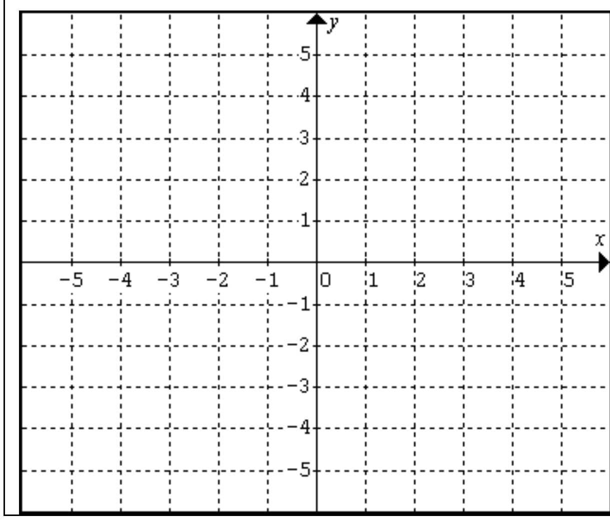
Some observations:

- To get a better plot of the graph, we would need to plot more points; choosing smaller increments of t would provide more coordinates (x, y) to create a smoother curve.
- The curve is not a single *function* where $y = f(x)$. It does not pass the vertical line test; some values for x have more than one corresponding value for y.
- The graph has an x-axis and a y-axis, but not a t-axis. This is because x and y are functions of t and t is not a function at all. The only indication of t (as time) is seen in the "bulleted" points and in most parametric curves, the t-values do not appear at all.
- The bulleted points, while calculated at equal time intervals from each other do not appear at equal distances from each other. This is because the "object", indicated by the points, speeds up or slows down as it moves through the path represented by the parametric equations.

Examples: Sketch the curve represented by each set of parametric equations.
Make a table of values (*t, x,* y), plot (*x,* y), and be sure to show the **orientation** (**direction**) of the curve.
Note: The **parameter** does not always have to be the variable *t*.

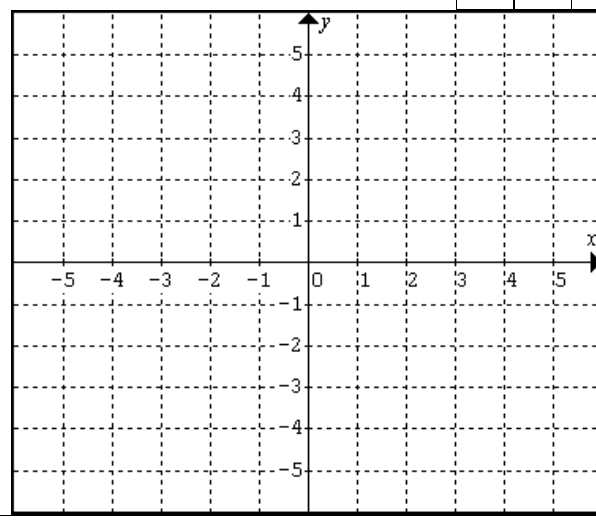
2. $x = 1 - 2t$, $y = 2 - t$ $\quad 0 \le t \le 3$

t	x	y



3. $x = 2\cos\theta$, $y = 2\sin\theta$ $\quad 0 \le \theta \le 2\pi$

θ	x	y



Eliminating the Parameter

Many curves that are represented by sets of parametric equations have graphs that can also be represented by rectangular equations (in *x* and *y*). This process is called **eliminating the parameter**. We use methods of elimination or substitution on the *system* of parametric equations to get rid of the variable *t* and create an equation that only has *x* and *y*.

Example 4:
Eliminate the parameter and write the rectangular equation.

$x = 1 - 2t$

$y = 2 - t$

Find the domain. We must limit the *x*-values (domain) to obtain *exactly* the same graph as before. So, using the original parameter, $0 \le t \le 3$, and the equation that relates *x* and *t*, **algebraically** find the domain for this graph.

Find the range. Sometimes just the “domain” can be misleading, so you can do the same thing with the *y*-values. Again, use the original parameter, $0 \le t \le 3$, and the equation that relates *y* and *t* to **algebraically** find the range for this graph.

Examples:

Eliminate the parameter and write the corresponding rectangular equation. Give the domain of the rectangular equation that would create the same curve as the parametric equations did.

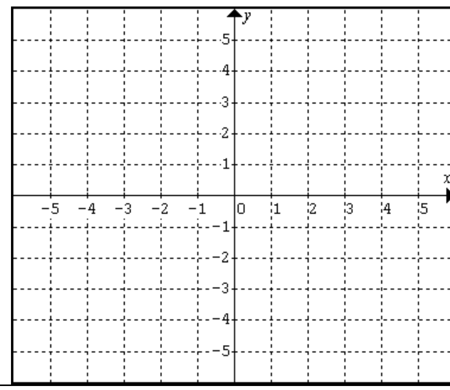
5. $x = t^2 - 4$, $y = \frac{t}{2}$ $\qquad -2 \le t \le 3$

Eliminate the parameter and write the rectangular equation.

Algebraically, find the domain.

Confirm your answers by graphing.

t	x	y



6. $x = 2\cos\theta$, $y = 2\sin\theta$ $\qquad 0 \le \theta \le 2\pi$

Eliminate the parameter and write the rectangular equation.

Hint: Square both sides of each equation, add the equations together, and look for a trigonometric identity that you can replace!

Algebraically, find the domain.

Note: Because of the periodic nature of the sin and cos functions, when **algebraically** solving for the **domain**, you must use half the parameter!

7. $x = -2 + 4\sin t$, $y = 7 + 3\cos t$ $\qquad 0 \le t \le 2\pi$

Eliminate the parameter and write the rectangular equation.

Think about what this equation would graph and tell the domain – do not solve algebraically.

Graphing Parametric Equations on the Graphing Calculator

- Change the MODE: Graph………Parametric (Param)
  Angle……..Radians (only necessary when we graph equations involving trigonometric functions)
  When you go to graph, you should see xt1 and yt1 to allow you to graph a parametric system.
- Change the WINDOW. You now must enter tmin, tmax, and tstep along with the x and y values.
- When graphing curves, do ZOOM SQUARE (SQR) to get a more accurate shape.

Example8:

Enter the equations that we graphed by hand on the first page.	Because the parameter is $-2 \le t \le 2$, enter these window values:		
$x = t^2$	$t\min = -2$	$x\min = -7$	$y\min = -5$
$y = t^2 - 4t$	$t\max = 2$	$x\max = 7$	$y\max = 12$
	$t\text{step} = 1$	$x\text{scl} = 1$	$y\text{scl} = 1$

Compare your graph to the hand drawn one. Then, change the ***t*step** to **.1** and graph, again.
What do you notice?

When graphing:

- The *t* values are determined by the parameter. You will have to determine the *x* and *y* min and max values (try a reasonable range and increase if necessary)
- If the parameter is *not* given, you must choose one that makes sense for your equations:
  Choose *t* values a few units on either side of zero, like $[-3,3]$.
  If *t* seems to indicate "time", choose *t* values from 0 to about 6.
  If *t* is an angle (θ), you can usually choose *t* values $[0,2\pi]$ - in RADIAN mode.
  Note: Even when the angle is given as θ, you must enter it as *t*.

Examples:
Graph using your calculator.

6. $\begin{array}{l} x = \sin(2t) \\ y = \cos(2t) \end{array}$ $\quad 0 \le t \le 2\pi$

Suggestions:

$t\text{step} = \dfrac{\pi}{24}$

$x = [-1,1] \quad y = [-1,1]$

Remember: ZoomSQR

7. $\begin{array}{l} x = 2(\cos\theta)^3 \\ y = 2(\sin\theta)^3 \end{array}$ $\quad 0 \le \theta \le 2\pi$

Suggestions:

$t\text{step} = \dfrac{\pi}{24}$

$x = [-2,2] \quad y = [-2,2]$

Remember: ZoomSQR

8. $\begin{array}{l} x = 12\cos\theta + 3\cos(6\theta) \\ y = 12\sin\theta - 3\sin(6\theta) \end{array}$ $\quad 0 \le \theta \le 2\pi$

Suggestions:

$t\text{step} = \dfrac{\pi}{24}$

$x = [-20,20] \quad y = [-20,20]$

Remember: ZoomSQR