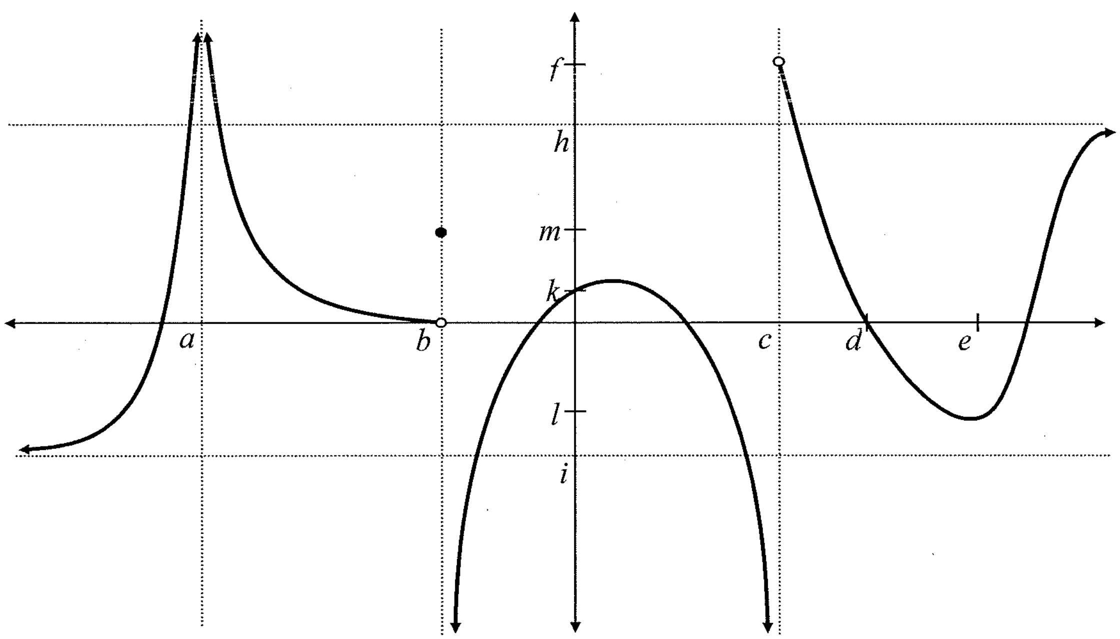
A.P. Calculus AB **Review of Limits** Answer Key

Refer to the graph of $g(x)$ below in order to answer the following questions. If a limit doesn't exist, explain why.

1. $\lim_{x\to\infty} g(x) = h$
2. $\lim_{x\to-\infty} g(x) = i$
3. $\lim_{x\to a^+} g(x) = \infty$
4. $\lim_{x\to a^-} g(x) = \infty$
5. $\lim_{x\to a} g(x) = \infty$
6. $\lim_{x\to 0} g(x) = K$
7. $\lim_{x\to b^+} g(x) = -\infty$
8. $\lim_{x\to b^-} g(x) = 0$
9. $\lim_{x\to b} g(x) =$ DNE b/c $\lim_{x\to b^-} g(x) \neq \lim_{x\to b^+} g(x)$
10. $\lim_{x\to c} g(x) =$ DNE b/c $\lim_{x\to c^-} g(x) \neq \lim_{x\to c^+} g(x)$
11. $\lim_{x\to d} g(x) = 0$
12. $\lim_{x\to e} g(x) = l$
13. $g(e) = l$
14. $g(0) = K$
15. $g(b) = m$



Find the values of the following limits if they exist.

16. $\lim_{x\to 9}\frac{3-\sqrt{x}}{9-x} = \lim_{x\to 9}\frac{3-\sqrt{x}}{(3+\sqrt{x})(3-\sqrt{x})} = \lim_{x\to 9}\frac{1}{3+\sqrt{x}} = \frac{1}{6}$

17. $\lim_{x\to -2}\frac{x^2+5x+6}{x^2-4} = \lim_{x\to -2}\frac{(x+2)(x+3)}{(x+2)(x-2)} = \lim_{x\to -2}\frac{x+3}{x-2} = -\frac{1}{4}$

18. $\lim_{x\to 4}\sqrt[3]{\frac{x^2-3x+4}{2x^2-x-1}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

19. $\lim_{x\to 2}\frac{4x+3}{3x-6} =$ DNE

$\lim_{x\to 2^-}\frac{4x+3}{3x-6} = -\infty$

$\lim_{x\to 2^+}\frac{4x+3}{3x-6} = \infty$

20. $\lim_{x\to 0}\frac{x}{\sqrt{x+3}-\sqrt{3}} \cdot \frac{(\sqrt{x+3}+\sqrt{3})}{(\sqrt{x+3}+\sqrt{3})}$

$= \lim_{x\to 0}\frac{x(\sqrt{x+3}+\sqrt{3})}{x+3-3}$

$= \lim_{x\to 0}\frac{x(\sqrt{x+3}+\sqrt{3})}{x} = \lim_{x\to 0}(\sqrt{x+3}+\sqrt{3})$

$= 2\sqrt{3}$

21. $\lim_{x\to -4}\frac{2x+8}{x-4} = \frac{0}{-8} = 0$

22. If $p(x)=\frac{3x-1}{9x^2-1}$ show that $p(\frac{1}{3})$ is undefined but $\lim_{x\to\frac{1}{3}} p(x)=\frac{1}{2}$

$p\left(\frac{1}{3}\right)=\frac{0}{0}$ $\quad \lim_{x\to\frac{1}{3}} \frac{3x-1}{9x^2-1}=\lim_{x\to\frac{1}{3}} \frac{3x-1}{(3x+1)(3x-1)}=\lim_{x\to\frac{1}{3}} \frac{1}{3x+1}=\frac{1}{2}$

23. Given that $g(x)=\begin{cases}3x-2, & x\neq\frac{1}{3}\\ 4, & x=\frac{1}{3}\end{cases}$ show that $g(\frac{1}{3})\neq\lim_{x\to\frac{1}{3}} g(x)$

$g\left(\frac{1}{3}\right)=4$

$\lim_{x\to\frac{1}{3}} g(x)=\lim_{x\to\frac{1}{3}}(3x-2)=-1$

Determine the values of the independent variable at which the function is discontinuous and tell why based on the definition.

24. $g(x)=\begin{cases}\frac{x^2+x-6}{x+3}, & x\neq-3\\ 1, & x=-3\end{cases}$

$g(-3)=1$

$\lim_{x\to-3} \frac{x^2+x-6}{x+3}=\lim_{x\to-3} \frac{(x+3)(x-2)}{x+3}$

$=\lim_{x\to3} x-2=-5$ disc cont @ $x=-3$ b/c $\lim_{x\to-3} g(x)\neq g(-3)$

25. $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ \sqrt{x}, & x>0\end{cases}$

$f(0)=0$

$\lim_{x\to0^-} f(x)=-1$ $\quad \lim_{x\to0^+} f(x)=0$

discont. at $x=0$ b/c $\lim_{x\to0} f(x)$ DNE

26. $g(x)=\frac{x^4-16}{x^2-4}=\frac{x^4-16}{(x+2)(x-2)}$

discont at $x=-2,2$

b/c $g(-2)$ and $g(2)$

are undefined

27. $h(x)=\begin{cases}\sqrt{-x}, & x<0\\ \sqrt[3]{x+1}, & x\geq0\end{cases}$

$h(0)=1$ $\quad \lim_{x\to0^-} h(x)=0$ $\quad \lim_{x\to0^+} h(x)=1$

discont at $x=0$ b/c

$\lim_{x\to0} h(x)$ DNE

Prove that the function is discontinuous at the number a. Determine if the discontinuity is removable or non-removable. IF it is removable, redefine $f(a)$ so that the discontinuity is removed. Show all work.

28. $f(x)=\frac{9x^2-4}{3x-2}$; at $a=\frac{2}{3}$

$f(x)$ is discont at $x=\frac{2}{3}$ b/c

$f\left(\frac{2}{3}\right)$ is undefined

$\lim_{x\to\frac{2}{3}} \frac{9x^2-4}{3x-2}=\lim_{x\to\frac{2}{3}} \frac{(3x+2)(3x-2)}{3x-2}$

$=\lim_{x\to\frac{2}{3}} 3x+2=4$ removable b/c

$\lim_{x\to\frac{2}{3}} f(x)$ exists

29. $f(t)=\begin{cases}9-t^2, & t\leq2\\ 3t+2, & t>2\end{cases}$; at $a=2$

$f(2)=5$

$\lim_{x\to2^-} f(x)=5$

$\lim_{x\to2^+} f(x)=8$

discont. at $x=2$

b/c $\lim_{x\to2} f(x)$ DNE

nonremovable b/c

30. $g(x) = \begin{cases} |x-3|, & x \neq 3 \\ 2, & x = 3 \end{cases}$; at $a = 3$

$g(3)=2$

$\lim_{x\to 3} |x-3| = 0$ discont at $x=3$

removable b/c $\lim_{x\to 3} g(x)$ exists.

b/c $\lim_{x\to 3} g(x) \neq g(3)$

31. $f(x) = \dfrac{3-\sqrt{x+9}}{x}$; at $a = 0$

$f(0) = \frac{0}{0}$ discont @ $x=0$ b/c $f(0)$ is undefined

$\lim_{x\to 0} \frac{3-\sqrt{x+9}}{x} \cdot \frac{3+\sqrt{x+9}}{3+\sqrt{x+9}}$

$= \lim_{x\to 0} \frac{9-(x+9)}{x(3+\sqrt{x+9})} = \lim_{x\to 0} \frac{-x}{x(3+\sqrt{x+9})}$

$= \lim_{x\to 0} \frac{-1}{3+\sqrt{x+9}} = \frac{-1}{6}$

removable b/c $\lim_{x\to 0} f(x)$ exists

Find all values of x for which the given function is continuous.

32. $h(x) = \dfrac{x+2}{x^2-7x+6}$

$= \frac{x+2}{(x-6)(x-1)}$

$(-\infty,1)\cup(1,6)\cup(6,\infty)$

33. $g(x) = \sqrt[4]{x^2+1}$

$(-\infty,\infty)$

33. $f(x) = \sqrt{9-x^2}$

$9-x^2 \geq 0$

$-3 \quad 3$

$[-3,3]$

34. $g(x) = \dfrac{x^3+7}{x^2-4} = \frac{x^3+7}{(x+2)(x-2)}$

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

35. $g(x) = \sqrt{\dfrac{x+4}{x-4}}$

$\frac{x+4}{x-4} \geq 0$ $(-\infty,-4]\cup(4,\infty)$

$+ \quad - \quad +$

$-4 \quad 4$

36. $f(x) = \sqrt{6x^2-x-2}$

$= \sqrt{(3x-2)(2x+1)}$

$+ \quad - \quad +$

$-\frac{1}{2} \quad \frac{2}{3}$

$(-\infty,-\frac{1}{2}]\cup[\frac{2}{3},\infty)$

37. $g(x) = \dfrac{|x|}{x}$

$(-\infty,0)\cup(0,\infty)$

38. $f(x) = \left(\dfrac{x^2}{x^2-4} - \dfrac{1}{x}\right)^{\frac{1}{3}}$

$(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

Find $(f \circ g)(x)$ and find all values of x for which $(f \circ g)(x)$ is continuous.

39. $f(x) = x^2$; $g(x) = x^2-3$

$f(g(x)) = (x^2-3)^2$

$(-\infty,\infty)$

40. $f(x) = \dfrac{1}{x-2}$; $g(x) = \sqrt{x}$

$f(g(x)) = \frac{1}{\sqrt{x}-2}$

$\sqrt{x}-2 \neq 0$

$\sqrt{x} \neq 2$

$x \neq 4, x \geq 0$

$[0,4)\cup(4,\infty)$

41. $f(x) = \sqrt{x+1}$; $g(x) = \sqrt[3]{x}$

$f(g(x)) = \sqrt{\sqrt[3]{x}+1}$

$\sqrt[3]{x}+1 \geq 0$

$- \quad +$

-1

$[-1,\infty)$

Find the values of c and/or k that make the function continuous on $(-\infty, \infty)$.

42. $f(x) = \begin{cases} kx-1, & x < 2 \\ kx^2, & x \geq 2 \end{cases}$

$f(2) = 4K$

$\lim_{x\to 2^-} f(x) = 2K-1$

$\lim_{x\to 2^+} f(x) = 4K$

$2K-1 = 4K$

$2K = -1$

$\boxed{K = \frac{-1}{2}}$

43. $g(x) = \begin{cases} x, & x \leq 1 \\ cx+k, & 1 < x < 4 \\ -2x, & x \geq 4 \end{cases}$

$g(1) = 1$

$\lim_{x\to 1^-} g(x) = 1$

$\lim_{x\to 1^+} g(x) = c+K$

$c+K = 1$

$g(4) = -8$

$\lim_{x\to 4^-} g(x) = 4c+K$

$\lim_{x\to 4^+} g(x) = -8$

$4c+K = -8$

$-c-K = -1$

$3c = -9$

$\boxed{c=-3 \\ K=4}$

Use the Intermediate Value Theorem (if it applies) to find a number c such that $f(c) = k$. (f and $[a, b]$ are given). If the theorem does not apply, explain why not.

44. $f(x) = \sqrt{25 - x^2}$; $[a, b] = [\cancel{-4.5}\ -5, 3]$; $k = 3$

$f(x)$ is cont on $[-5, 3]$ ✓

$f(-5) = 0$ $\quad 0 \le 3 \le 4$ ✓

$f(3) = 4$

$\sqrt{25 - x^2} = 3$

$25 - x^2 = 9$

$x^2 = 16$

$x = \pm 4$

$\boxed{c = -4}$

45. $f(x) = \begin{cases} 1 + x & , -4 \le x \le -2 \\ 2 - x & , -2 < x \le 1 \end{cases}$; $[a, b] = [-4, 1]$; $k = ½$

$\lim\limits_{x \to -2^-} f(x) = -1$ $\quad \lim\limits_{x \to -2^+} f(x) = 3$ $\quad \therefore$ IVT does not apply b/c $f(x)$ is not cont on $[-4, 1]$

46. $f(x) = \dfrac{5}{2x - 1}$; $[a, b] = [\cancel{0, 1}\ 1, 2]$; $k = 2$

$f(x)$ is cont on $[1, 2]$ ✓

$f(1) = 5$ $\quad \frac{5}{3} \le 2 \le 5$ ✓

$f(2) = \frac{5}{3}$

$\dfrac{5}{2x - 1} = 2$

$5 = 4x - 2$

$4x = 7$

$x = \frac{7}{4}$

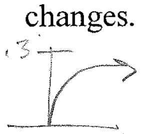
$\boxed{c = \frac{7}{4}}$

The proportion of pine trees infested with pine beetles is approximated by the equation $y = \dfrac{.26x}{x + 34.83}$, where x is the beetle density measured by the average number of beetles per tree.

47. As the density of the beetles increases, complete the table below:

x	10	50	100	500	1000	10,000	100,000
y	.058	.15325	.19284	.24307	.25125	.2591	.25991

48. Using a calculator graph this function and describe the behavior of the infestation as the density changes.



the infestation grows quickly at first but then levels off

49. Discuss the rate of change as the beetle density increases.

the rate of change decreases as the density increases

50. According to this model, as the pine beetle density increases without bound, what proportion of trees is infected? Defend your work algebraically using limits.

$\lim\limits_{x \to \infty} \dfrac{.26x}{x + 34.83} = .26$

about 26% of trees will be infected as the density increases w/out bound