Hey everybody, It's Warmup Time!!

Oh the memories! Evaluate each integral below.

1) $\int \tan^4 x\, dx =$

2) $\int \sin^3(4x)\, dx =$

3) $\int x^2 e^{3x} dx =$

4) $\int x\cos(x^2)\, dx =$

1) $\int \tan^4 x\, dx = \int \tan^2 x\,(\sec^2 x - 1)dx$

✓ $= \int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx$

$= \int u^2 du - \int (\sec^2 x - 1)dx$

$= \frac{1}{3}\tan^3 x - \tan x + x + C$

2) $\int \sin^3(4x)\, dx = \int \sin(4x)\,(1 - \cos^2(4x))dx$

✓ $= \int \sin(4x)\, dx - \int \sin(4x)\cos^2(4x)\, dx$

$= -\frac{1}{4}\cos(4x) - \int -\frac{1}{4}u^2 du$

$= -\frac{1}{4}\cos(4x) + \frac{1}{12}\cos^3(4x) + C$

✓ 3) $\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

✓ 4) $\int x\cos(x^2)\, dx = \int \frac{1}{2}\cos u\, du = \frac{1}{2}\sin u + C = \frac{1}{2}\sin(x^2) + C$

(3)

	u	dv
+	x^2	e^{3x}
−	$2x$	$\frac{1}{3}e^{3x}$
+	2	$\frac{1}{9}e^{3x}$
−	0	$\frac{1}{27}e^{3x}$

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Question 4

Let's get our brain going with this BC problem from 2015...

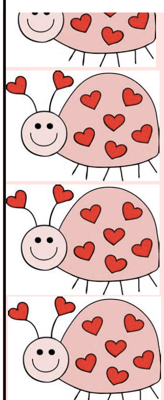
Consider the differential equation $\frac{dy}{dx} = 2x - y$.

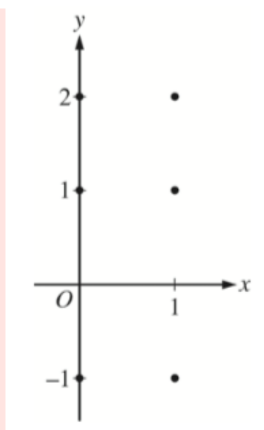
(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.





(23) $\int_0^\infty e^{-x}\cos x\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\cos x\,dx$

$\int e^{-x}\cos x\,dx = e^{-x}\sin x + \int e^{-x}\sin x\,dx$

$u = e^{-x}$ $du = -e^{-x}\,dx$

$v = \sin x$ $dv = \cos x\,dx$

$u = e^{-x}$ $du = -e^{-x}\,dx$

$v = -\cos x$ $dv = \sin x\,dx$

$\int e^{-x}\cos x\,dx = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx$

$2\int e^{-x}\cos x\,dx = e^{-x}\sin x - e^{-x}\cos x$

so $\int e^{-x}\cos x\,dx = \frac{1}{2}\left(e^{-x}\sin x - e^{-x}\cos x\right)$

$\lim_{b\to\infty} \int_0^b e^{-x}\cos x\,dx = \lim_{b\to\infty}\left[\frac{1}{2}\left(e^{-x}\sin x - e^{-x}\cos x\right)\right]\Big|_0^b$

$= \lim_{b\to\infty}\left[\frac{1}{2}\left(e^{-b}\sin b - e^{-b}\cos b\right) - \frac{1}{2}\left(e^{0}\sin 0 - e^{0}\cos 0\right)\right]$

$= \lim_{b\to\infty}\left[\frac{1}{2}\frac{\sin b}{e^b} - \frac{1}{2}\frac{\cos b}{e^b} - 0 + \frac{1}{2}(1)\right] = 0 - 0 - 0 + \frac{1}{2} = \boxed{\frac{1}{2}}$

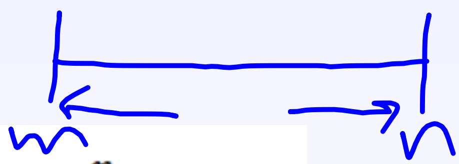
8.8b Improper Integrals Part 2!

At the end of this lesson you will be able to:

- Evaluate integrals containing vertical asymptotes within the limits

Yesterday the improper integrals were easy to recognize because one (or both) of the limits contained ∞.

Suppose the limits are finite but there is an infinite discontinuity (vertical asymptote) within the interval. What then?



Case 1:

(VA at x = m)

$$\int_m^n f(x)dx = \lim_{a \to m^+} \int_a^n f(x)dx$$

Case 2:

(VA at x = n)

$$\int_m^n f(x)dx = \lim_{b \to n^-} \int_m^b f(x)dx$$

Case 3:

(VA in between m and n)

$$\int_m^n f(x)dx = \lim_{a \to k^-} \int_m^a f(x)dx + \lim_{a \to k^+} \int_a^n f(x)dx$$

Remember, you only have to 'worry' about recognizing these if you are dealing with a definite integral problem.

So, from now on, whenever you have a definite integral problem you will need to check for one of the following before proceding:

1) Is ∞ one of the limits? (easy to spot)

2) Is there an x-value at or between the limits that causes division by zero? (need to think about)

ex) $\int_0^1 \frac{1}{x^{\frac{1}{3}}}\,dx = \lim_{a \to 0^+} \int_a^1 \frac{1}{x^{\frac{1}{3}}}\,dx$

$= \lim_{a \to 0^+} \left[\frac{3}{2} x^{\frac{2}{3}} \Big|_a^1 \right]$

$= \lim_{a \to 0^+} \left[\frac{3}{2} - \frac{3}{2} a^{\frac{2}{3}} \right] = \frac{3}{2}$

Give this one a try!

$$\int_0^{27} \frac{dx}{\sqrt[3]{27-x}} = \lim_{b\to 27^-} \int_0^{b} \frac{dx}{\sqrt[3]{27-x}}$$

$$= \lim_{b\to 27^-} \left(-\frac{3}{2}(27-x)^{\frac{2}{3}}\Big|_0^b\right)$$

$$= \lim_{b\to 27^-} \left(-\frac{3}{2}(27-b)^{\frac{2}{3}} - -\frac{27}{2}\right)$$

$$= 0 + \frac{27}{2} = \frac{27}{2}$$

Hmmm, how about this one?

$$\int_0^1 \frac{1}{x^3}dx = \lim_{a\to 0^+}\int_a^1 \frac{1}{x^3}dx$$

$$= \lim_{a\to 0^+} -\frac{1}{2x^2}\Big|_a^1$$

$$= \lim_{a\to 0^+}\left(-\frac{1}{2} - -\frac{1}{2a^2}\right)$$

$$= -\frac{1}{2} + \infty = \infty$$

Just one more notch, you're almost there!

$$\int_0^3 (x-1)^{-\frac{2}{3}}dx = \lim_{a\to 1^-}\int_0^a (x-1)^{-\frac{2}{3}}dx + \lim_{a\to 1^+}\int_a^3 (x-1)^{-\frac{2}{3}}dx$$

$$= \lim_{a\to 1^-}\left(3(x-1)^{\frac{1}{3}}\Big|_0^a\right) + \lim_{a\to 1^+}\left(3(x-1)^{\frac{1}{3}}\Big|_a^3\right)$$

$$= \lim_{a\to 1^-}\left(3(a-1)^{\frac{1}{3}} - -3\right) + \lim_{a\to 1^+}\left(3\sqrt[3]{2} - 3(a-1)^{\frac{1}{3}}\right)$$

$$= 0 + 3 + 3\sqrt[3]{2} - 0 = 3 + 3\sqrt[3]{2}$$

Do you remember?

$$\int_{2}^{\infty} \frac{1}{\sqrt{x-1}}\,dx = \lim_{b\to\infty} \int_{2}^{b} \frac{1}{\sqrt{x-1}}\,dx$$

$$= \lim_{b\to\infty} \left(2\sqrt{x-1}\Big|_{2}^{b}\right)$$

$$= \lim_{b\to\infty} \left(2\sqrt{b-1} - 2\right)$$

$$= \infty - 2 = \infty$$

One more type of problem we haven't really discussed but need to include in this unit.

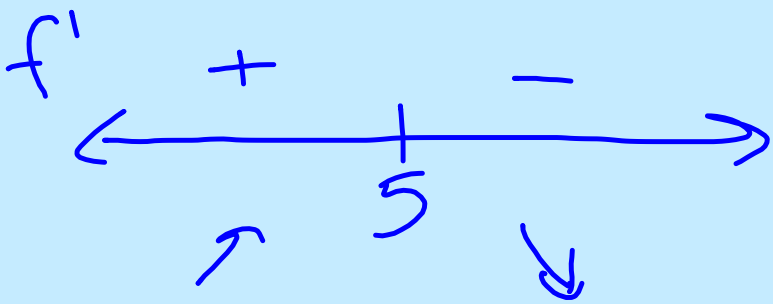
$$\int \frac{5}{1+\sin x}dx = 5\int \frac{1}{1+\sin x}dx \cdot \frac{1-\sin x}{1-\sin x}$$

$$= 5\int \frac{1-\sin x}{1-\sin^2 x}dx$$

$$= 5\int \frac{1-\sin x}{\cos^2 x}dx$$

$$= 5\int (\sec^2 x - \sec x \tan x)dx$$

$$= 5\tan x - 5\sec x + C$$



An abs max occurs @ $x = 5$ b/c

→ f' chgs from + to − @ $x = 5$ and $x = 5$ is the only critical #

→ f' is > 0 for all $x < 5$ and $f' < 0$ for all $x > 5$

What have we learned?

- What is an improper integral?
- What wonderful things do I use to rewrite improper integrals in a form that can be integrated?