Warmup! Work together on the following and write your answers on your whiteboards.

10. $\int_1^4 t^{-3/2}\, dt =$

(A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

---

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x\to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

---

12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

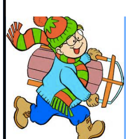
(A) $(1, -5)$ is a local maximum of f.

(B) $(1, -5)$ is a point of inflection of the graph of f.

(C) $(-1, -1)$ is a local maximum of f.

(D) $(-1, -1)$ is a local minimum of f.

(E) $(-1, -1)$ is a point of inflection of the graph of f.

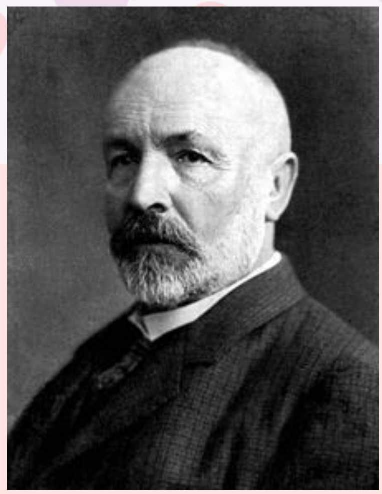


Think About It!!

What does ∞ - ∞ equal?

Discuss with your group for 1 minute.

Follow up: Which mathematician did groundbreaking work to describe the various degrees of infinity? Hint: his pictures (young and old) are below.



Georg Cantor was born in Russia in 1845 but grew up in Germany. He is the founder of all set theory and was the first to introduce the concept of different 'sizes' of infinity. He began to label sets as either finite, denumerable infinite, or nondenumerable infinite. A set was considered denumerable if it could be mapped in a 1-1 relationship with the set of natural numbers. So the sets of naturals, rationals, and ordered pairs are denumerable, while the set of real numbers is nondenumerable. His work on ordinal and cardinal numbers and the cardinality of sets is phenomenal, but he was constantly laughed at and dismissed by his colleagues, so he ended up suffering from constant depression throughout his life. If you look up his Wikipedia and start clicking on all of the links within his page, your time would not be wasted. A deep study of set theory will make your brain hurt (and hence grow)!! I love the diagonal theory he came up with later on in life, mostly because I can understand it! :) ✓

8.7b Special Cases of L'Hopital's Rule

Learning target:

- I can use L'Hopital's Rule to evaluate limits in indeterminate form

So far, we have applied L'Hopital's Rule to limits of the form 0/0 or ∞/∞.

What if we get a limit of the type: 0^0, $0 \cdot \infty$, 1^∞ or $\infty - \infty$?

These are all still considered indeterminate forms, but cannot directly have the rule applied.

Luckily, we can often rewrite the limits in a form we can use.

Some limits just need to be slightly rewritten in order for L'Hopital's Rule to apply. Give these a try!

$0 \cdot \infty$

1) $\lim_{x\to\infty} e^{-x}\sqrt{x} = \lim_{x\to\infty} \frac{\sqrt{x}}{e^x}$

$= \lim_{x\to\infty} \frac{1}{2e^x\sqrt{x}} = 0$

$\lim_{x\to\infty} \sqrt{x} = \infty = \lim_{x\to\infty} e^x$

therefore L'Hopital applies

OR $= 0\ (using\ comparitive\ growth\ rates)$

$\infty - \infty$

2) $\lim_{x\to 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

$= \lim_{x\to 1^+} \frac{x-1-\ln x}{\ln x\,(x-1)}$

$\lim_{x\to 1^+} x-1-\ln x = 0 = \lim_{x\to 1^+} \ln x\,(x-1)$

therefore L'Hopital applies

$= \lim_{x\to 1^+} \frac{1-\frac{1}{x}}{\frac{x-1}{x}+\ln x} \cdot \frac{x}{x}$

$= \lim_{x\to 1^+} \frac{x-1}{x-1+x\ln x}$

$\lim_{x\to 1^+} x-1 = 0 = \lim_{x\to 1^+} (x-1+x\ln x)$

therefore L'Hopital applies

$= \lim_{x\to 1^+} \frac{1}{1+\ln x+1} = \frac{1}{2}$

Do you remember when...

If $y = x^{2x+3}$ find dy/dx

✓

$$\ln y = (2x+3)\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + \frac{2x+3}{x}$$

$$\frac{dy}{dx} = \left(2\ln x + \frac{2x+3}{x}\right)x^{2x+3}$$

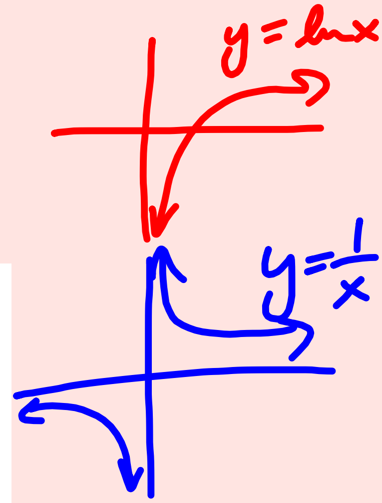
4) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2}} = \lim_{x\to\infty} \frac{x}{x} = 1$

remember, $\sqrt{x^2} = x$ if x > 0 and $\sqrt{x^2} = -x$ if x < 0

Can you take the same derivative technique and apply it to the following limit? Try it and see!

0^0

$$\lim_{x\to 0^+} (\sin x)^x$$



$$y = \lim_{x\to 0^+} (\sin x)^x$$

$0 \cdot -\infty$

$$\ln y = \lim_{x\to 0^+} (x \ln(\sin x))$$

$$\ln y = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}}$$

$\lim_{x\to 0^+} \ln(\sin x) = -\infty$ *and* $\lim_{x\to 0^+} \frac{1}{x} = \infty$

therefore L'Hopital applies

$$\ln y = \lim_{x\to 0^+} -x^2 \cot x = \lim_{x\to 0^+} \frac{-x^2}{\tan x}$$

$\lim_{x\to 0^+} -x^2 = 0 = \lim_{x\to 0^+} \tan x$

therefore L'Hopital applies

$$\ln y = \lim_{x\to 0^+} -2x\cos^2 x = 0$$

$$y = e^0 = 1$$

$$\lim_{x\to 0^+} (\sin x)^x = 1$$

How about this one?

$$\lim_{x \to 4^+} [3(x-4)]^{x-4}$$

$$y = \lim_{x \to 4^+} [3(x-4)]^{x-4}$$

$$\ln y = \lim_{x \to 4^+} (x-4)\ln(3(x-4))$$

$$\ln y = \lim_{x \to 4^+} \frac{\ln 3 + \ln(x-4)}{\frac{1}{x-4}}$$

$$\lim_{x \to 4^+} (\ln 3 + \ln(x-4)) = -\infty \ \ and \ \ \lim_{x \to 4^+} \frac{1}{x-4} = \infty$$

therefore L'Hopital applies

✓

$$\ln y = \lim_{x \to 4^+} \frac{\frac{1}{x-4}}{-\frac{1}{(x-4)^2}} = \lim_{x \to 4^+} (4-x) = 0$$

$$y = e^0 = 1$$

$$\lim_{x \to 4^+} [3(x-4)]^{x-4} = 1$$

Brain Break! It's 'Getting to Know You' Tuesday!

Please write the full solution to the following(along with any required justification) on your whiteboards.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$$y=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$\infty \cdot 0$

$$\ln y=\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)=\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)=0=\lim_{x\to\infty}\frac{1}{x}$$

therefore L'Hopital applies

✓

$$\ln y=\lim_{x\to\infty}\frac{\frac{-\frac{1}{x^2}}{1+\frac{1}{x}}}{-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}=1$$

$$y=e \text{ so } \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

Note: You need to know the difference between indeterminate and determinate

Indeterminate	Determinate
0/0 - L'Hopital	$0^{\infty} = 0$
∞/∞ - L'Hopital	$0^{-\infty} = 0$
0^0 - use logs to rewrite	∞ + ∞ = ∞
1^{∞} - use logs to rewrite	-∞ - ∞ = -∞
$0 \cdot \infty$ - rewrite as 0/0 or ∞/∞	
∞ - ∞ - use algebra to combine	

Let's do some review!

2006 BC #4

What have we learned?

- Can I rewrite indeterminate limits so that L'Hopital can be applied?