4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\, dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right) dx$

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2 - 3\right)^5 dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\, du$

(B) $\int_{-2}^{13} u^5\, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5\, du$

(D) $\int_{-1}^{4} u^5\, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5\, du$

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1 - x^2}}$

(B) $\frac{xy}{\sqrt{1 - x^2}}$

(C) $\frac{y}{1 + x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1 + x^2}$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$$\int_4^{15} R(t)\,dt \approx 3(6.2) + 5(5.9) + 3(5.6)$$

$$+50$$

It's Warmup Time!!

Let's go WAYYY back! Evaluate the following:

1) $\lim_{x \to 2} \frac{4x-8}{3x-2}$ ✓ = 0

2) $\lim_{x \to 0} \frac{\frac{1}{5x+2} - \frac{1}{2}}{x}$ = -5/4

3) $\lim_{x \to 5} \frac{x^2 - 10}{x - 5}$ = DNE

4) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$ = 1/2

5) $\lim_{x \to \infty} \frac{3x^2 + 1}{2x - 5}$ = ∞

6) $\lim_{x \to \infty} \frac{3x + 1}{5 - 2x}$ = -3/2

8.7a L'Hopital's Rule

Learning target:

- I can use L'Hopital's Rule to evaluate limits in indeterminate form

Guillaume Francois Antoine Marquis de l'Hopital

was a French mathematician who lived from 1661 to 1704. In 1691 he became friends with Johann Bernoulli who tutored him privately about infinitesimal calculus. In 1694, l'Hopital made a deal with Bernoulli: he would pay Bernoulli 300 Francs a year and in return Bernoulli would tell l'Hopital all of his mathematical discoveries instead of publishing them himself.

L'Hopital published 'his' first book in 1696 which dealt with differential calculus and applications related to differential geometry (no integrals at all). After l'Hopital died, Bernoulli tried to convince the world the the majority of the book was his work, but very few believed him until someone found a manuscript of Bernoulli's in the Basel University library dating to 1691 that was extremely similar to l'Hopital's text. Too bad the manuscript was not discovered until 1921. :(

L'Hopital's Rule!

Suppose you are asked $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$

If $\lim\limits_{x \to a} f(x) = 0 = \lim\limits_{x \to a} g(x)$

OR if $\lim\limits_{x \to a} f(x) = \pm\infty$ and $\lim\limits_{x \to a} g(x) = \pm\infty$

$\therefore$ L'Hopital applies

then $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to c} \dfrac{f'(x)}{g'(x)}$

Note #1: do NOT use quotient rule. It's just the derivative of the top over the derivative of the bottom.

Note #2: EVERY TIME you apply L'Hopital's rule, you MUST justify its use to receive credit

Let's re-try some of those warmup derivatives!
Be careful! L'Hopital doesn't always apply!
Note: every time you use L'Hopital's Rule, you must justify its use

1) $\lim_{x\to 2} \frac{4x-8}{3x-2}$ $= 0$ (not L'Hopital)

2) $\lim_{x\to 0} \frac{\frac{1}{5x+2}-\frac{1}{2}}{x}$

$\lim_{x\to 0} \frac{1}{5x+2}-\frac{1}{2} = 0 = \lim_{x\to 0} x$ $\therefore$ L'Hopital applies

$= \lim_{x\to 0} \frac{(5x+2)^{-1}-\frac{1}{2}}{x} = \lim_{x\to 0} \frac{-(5x+2)^{-2}(5)}{1}$

$= \lim_{x\to 0} \frac{-5}{(5x+2)^2} = \frac{-5}{4}$

3) $\lim_{x\to 5} \frac{x^2-10}{x-5}$ $= \frac{15}{0} \Rightarrow$ DNE

4) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

$\lim_{x\to 0}(\sqrt{x+1}-1) = 0 = \lim_{x\to 0} x$ $\therefore$ L'Hopital applies

$= \lim_{x\to 0} \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}}{1} = \lim_{x\to 0} \frac{1}{2\sqrt{x+1}} = \frac{1}{2}$

5) $\lim_{x\to\infty} \frac{3x^2+1}{2x-5}$

$\lim_{x\to\infty}(3x^2+1) = \infty = \lim_{x\to\infty}(2x-5)$ $\therefore$ L'Hopital applies

$= \lim_{x\to\infty} \frac{6x}{2} = \infty$

6) $\lim_{x\to\infty} \frac{3x+1}{5-2x}$

$\lim_{x\to\infty}(3x+1) = \infty$ $\lim_{x\to\infty}(5-2x) = -\infty$ $\therefore$ L'Hopital applies

$= \lim_{x\to\infty} \frac{3}{-2} = \frac{3}{-2}$

$\lim_{x\to 0} \sin x = 0 = \lim_{x\to 0} x$ $\therefore$ L'Hopital applies

ex] $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$

Sometimes you might have to apply the rule more than once:

$$\lim_{x\to 3}\frac{x^3-4x^2-3x+18}{x^3-x^2-21x+45}$$

$\lim_{x\to 3}(x^3-4x^2-3x+18) = 0 = \lim_{x\to 3}(x^3-x^2-21x+45)$

$\therefore$ L Hopital applies

$$= \lim_{x\to 3}\frac{3x^2-8x-3}{3x^2-2x-21}$$

$\lim_{x\to 3}(3x^2-8x-3) = 0 = \lim_{x\to 3}(3x^2-2x-21)$

$\therefore$ L Hopital applies

$$= \lim_{x\to 3}\frac{6x-8}{6x-2} = \frac{10}{16} = \frac{5}{8}$$

More examples!

Determine which of the following can be evaluated using L'Hopital's Rule. If the rule applies, use it. If not, evaluate the limit using other means.

1) $\lim_{x\to 1} \dfrac{\arctan x - \frac{\pi}{4}}{x-1}$

$\lim_{x\to 1}\left(\arctan x - \frac{\pi}{4}\right) = 0 = \lim_{x\to 1}(x-1)$ ∴ LHopital applies

$= \lim_{x\to 1} \dfrac{\frac{1}{1+x^2}}{1} = \dfrac{1}{2}$

2) $\lim_{x\to\infty} \dfrac{x^2}{e^x}$

$\lim_{x\to\infty} x^2 = \infty = \lim_{x\to\infty} e^x$ and $\lim_{x\to\infty} 2x = \infty = \lim_{x\to\infty} e^x$

$= \lim_{x\to\infty} \dfrac{2x}{e^x} = \lim_{x\to\infty} \dfrac{2}{e^x} = 0$

∴ L'Hopital applies twice!

OR $\lim_{x\to\infty} \dfrac{x^2}{e^x} = 0$ by comp. growth rates

3) $\lim_{x\to\infty} \dfrac{x^2}{\sqrt{x^2+1}}$

$\lim_{x\to\infty} x^2 = \infty = \lim_{x\to\infty} \sqrt{x^2+1}$ ∴ LHopital applies

$= \lim_{x\to\infty} \dfrac{2x}{\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)} = \lim_{x\to\infty} \dfrac{2\sqrt{x^2+1}}{1} = \infty$

or $= \infty$ by comp growth rates

4) $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \dfrac{1}{\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)} = \lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x}$

$= \lim_{x\to\infty} \dfrac{\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)}{1} = \lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$ Oh No!!

4) $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \dfrac{x}{\sqrt{x^2}} = \lim_{x\to\infty} \dfrac{x}{x} = 1$

remember, $\sqrt{x^2} = x$ if $x > 0$ and $\sqrt{x^2} = -x$ if $x < 0$

February 5, 2018

The AP Exam is 3 months and 10 days away. Are you ready???

This is from the AP BC Exam from 2013 - No Calculators

Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x \to 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

February 5, 2018

Just for fun! 3 Review Integrals!!!

1) $$\int x^3(1-x^2)^5dx$$

$$\int x^3\left(\sqrt{1-x^2}\right)^{10}dx$$

2) $$\int \frac{3x^4}{x^2-4}dx$$

3) $$\int \frac{x^2+4}{x^2+4x}dx$$

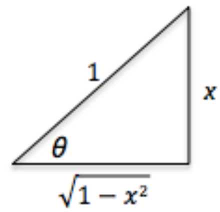
1)

$\sqrt{1-x^2}=\cos\theta$

$x=\sin\theta$

$dx=\cos\theta\, d\theta$

$x^3=\sin^3\theta$



$$\int \sin^3\theta\cos^{10}\theta\cdot\cos\theta\, d\theta$$

$$\int \sin^3\theta\cos^{11}\theta\, d\theta$$

$$\int \sin\theta\cos^{11}\theta\,(1-\cos^2\theta)d\theta$$

$$\int (\sin\theta\cos^{11}\theta-\sin\theta\cos^{13}\theta)d\theta$$

$$-\frac{1}{12}\cos^{12}\theta+\frac{1}{14}\cos^{14}\theta+C$$

$$-\frac{(1-x^2)^6}{12}+\frac{(1-x^2)^7}{14}+C$$

or

$u=1-x^2, so\ du=\ -2xdx$

$$\int -\frac{1}{2}(1-u)u^5du=\int -\frac{1}{2}(u^5-u^6)du$$

$$=-\frac{1}{12}u^6+\frac{1}{14}u^7+C=\frac{(1-x^2)^7}{14}-\frac{(1-x^2)^6}{12}+C$$

2)

$$=\int\left(3x^2+12+\frac{48}{x^2-4}\right)dx$$

$$=x^3+12x+48\int\frac{1}{x^2-4}dx$$

$$=x^3+12x+\int\frac{48}{(x+2)(x-2)}dx$$

$$=x^3+12x+\int\left(\frac{12}{x-2}-\frac{12}{x+2}\right)dx$$

$$=x^3+12x+12\ln|x-2|-12\ln|x+2|+C$$

3)

$$=\int\left(1+\frac{4-4x}{x^2+4x}\right)dx$$

$$=x+\int\frac{4-4x}{x(x+4)}dx$$

$$=x+\int\left(\frac{1}{x}-\frac{5}{x+4}\right)dx$$

$$=x+\ln|x|-5\ln|x+4|+C$$

What have we learned?

- What does a limit need to 'equal' at first in order to apply L'Hopital's Rule?
- Can I correctly apply the rule?