1) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$

$\lim_{n\to\infty} \frac{n^2-1}{n^2+n} = 1$

div. by n^{th} term

2) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln n}}$

$u = \ln x$

$du = \frac{1}{x}\,d\cancel{x}$

$\lim_{b\to\infty} \int_1^b u^{-\frac{1}{2}}\,du = \lim_{b\to\infty} \left(2u^{\frac{1}{2}} \Big|_1^b \right) = \infty - 2$

div. by integral test

3) $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n}{2}\right)}{n^2+4n}$

$\frac{\cos\left(\frac{n}{2}\right)}{n^2+4n} \le \frac{1}{n^2}$

conv by p test

Conv by direct comp

4) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)}{n^2+n}$

conv by

$\displaystyle\lim_{n\to\infty} \frac{n-1}{n^2+n} = 0$ alt series tst

5) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

$\displaystyle\lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{n}$

$\displaystyle= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} = e^{-1} = \frac{1}{e} < 1$

converges by root test

6) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$ $\displaystyle\lim_{n\to\infty} \left| \frac{\frac{n-1}{n^2+n}}{\frac{1}{n}} \right|$

$$= \lim_{n\to\infty} \left| \frac{n(n-1)}{n^2+n} \right| = 1$$

$\therefore$ diverges by limit comp.

7) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{2\cdot 5\cdot 8\cdot \ldots \cdot (3n+2)}$ $1\cdot 2\cdot 3\cdot \ldots \cdot 9\cdot 10$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)!}{2\cdot 5\cdot 8 \cdots (3n+2)(3(n+1)+2)}}{\frac{n!}{2\cdot 5\cdot 8\cdot \ldots \cdot (3n+2)}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\overset{(n+1)\cancel{n!}}{(n+1)!}\,\cancel{[2\cdot 5\cdot 8\cdot \ldots \cdot (3n+2)]}}{\cancel{n!}\,[\cancel{2\cdot 5\cdot 8\cdot \ldots \cdot (3n+2)}(3n+5)]} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n+1}{3n+5} \right| = \frac{1}{3} < 1$$

conv. by ratio test

8) **Find the sum:** $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

$$= \sum_{n=1}^{\infty} \frac{(-3)(-3)^n}{8^n} = \sum_{n=1}^{\infty} (-3)\left(\frac{-3}{8}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{9}{8}\left(\frac{-3}{8}\right)^n = \frac{\frac{9}{8}}{1-\frac{-3}{8}} = \frac{9}{11}$$

9) **Find the sum:** $\sum_{n=3}^{\infty} \frac{1}{n^2+n}$

$$\sum_{n=3}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

$$= \frac{1}{3} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{6}} + \cdots$$

$$= \frac{1}{3} - \lim_{n\to\infty} \frac{1}{n+1} = \frac{1}{3}$$

10) Express this repeating decimal as a fraction: $0.64\overline{81}$

$$.64 + \sum_{n=0}^{\infty} .0081\left(\frac{1}{100}\right)^n$$

$$= \frac{64}{100} + \frac{.0081}{1-\frac{1}{100}}$$

$$= \frac{64}{100} + \frac{.0081}{\frac{99}{100}} = \frac{64}{100} + \frac{81}{9900}$$

$$= \frac{6417}{9900} = \frac{713}{1100}$$

11) Find the power series, centered at c = 2, for $f(x) = \frac{3}{2x-1}$

$$\frac{3}{2(x-2)-1+4} = \frac{3}{2(x-2)+3} = \frac{3}{3+2(x-2)}$$

$$= \frac{1}{1 - -\frac{2}{3}(x-2)} = \sum_{n=0}^{\infty}\left(-\frac{2}{3}(x-2)\right)^n$$

$$= \sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n (x-2)^n$$

12) Determine the interval of convergence of the series found in #11.

$$\left|-\frac{2}{3}(x-2)\right| < 1$$

$$|x-2| < \frac{3}{2} \qquad -\frac{3}{2} < x-2 < \frac{3}{2}$$

$$\frac{1}{2} < x < \frac{7}{2}$$

13) Write a Taylor polynomial with 4 nonzero terms centered at c = 0 for $f(x) = \cos x$

$f(x) = \cos x \qquad f(0) = 1$

$f'(x) = -\sin x \qquad f'(0) = 0$

$f''(x) = -\cos x \qquad f''(0) = -1$

$f'''(x) = \sin x \qquad f'''(0) = 0 \quad \ldots$

$\vdots$

$$P(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

14) Write the power series for $f(x) = \cos x$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

15) Use the series found in #14 to create a power series for $f(x) = \sin\sqrt{x}$

$$\sin x = \int \cos x \, dx$$

$$= \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!(2n+1)}$$

$$\sin\sqrt{x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{\frac{2n+1}{2}}}{(2n)!(2n+1)}$$

16) Use the work from #14 and #15 to approximate $\int_0^1 \sin\sqrt{x}\,dx$ with an error less than 0.00001. Write your answer to at least 8 decimal places.

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{\frac{2n+1}{2}}}{(2n)!(2n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+\frac{3}{2}}}{(2n)!(2n+1)(n+\frac{3}{2})} \Bigg|_0^1$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!(2n+1)(n+\frac{3}{2})}$$

$$a_4 < .00001$$

$$\sum_{n=0}^{3} Y1 \approx .6023368606 7$$

actual value $\approx .6023373578 8$

March 25, 2016