Chapter 3 Review

1. Find the differential, dy, of $y = -2\sec^3(\sin(4x))$

2. Complete two iterations of Newton's Method to approximate $3^{1/4}$ starting with a value of 1

3. Find the absolute extrema of $f(x) = 2x - 3x^{2/3}$ on [-1, 3]

4. A container in the shape of a right circular cylinder with no top has a surface area of 3π ft^2. What height, h, and radius, r, will maximize the volume of the cylinder?

5. Find all values of c that satisfy the mean value theorem for derivatives for the function $y = 2x^3 - 3x$ on [-1, 2]

6. Given the function $y = x^4 - 5x^2 - 3x + 1$, identify the x- and y-intercepts, the increasing and decreasing intervals, the relative extrema, the intervals of concavity, and the points of inflection. (You may need to use your calculator for the "solving" feature)

Chapter 3 Review

1. Find the differential, dy, of y = -2sec^3(sin(4x))

$$dy = -2(3)\sec^2(\sin(4x))\sec(\sin(4x))\tan(\sin(4x))\cos(4x)(4)\,dx$$
$$= -24\sec^3(\sin(4x))\tan(\sin(4x))\cos(4x)\,dx$$

2. Complete two iterations of Newton's Method to approximate $3^{1/4}$ starting with a value of 1

$x = \sqrt[4]{3}$

$x^4 = 3$

$y = x^4 - 3$

$y' = 4x^3$

$y(1) = -2$

$y'(1) = 4$

$y + 2 = 4(x - 1)$

$x = \frac{1}{2} + 1 = \frac{3}{2}$

$y\left(\frac{3}{2}\right) = \frac{81}{16} - \frac{48}{16} = \frac{33}{16}$

$y'\left(\frac{3}{2}\right) = 4\left(\frac{27}{8}\right) = \frac{27}{2}$

$y - \frac{33}{16} = \frac{27}{2}\left(x - \frac{3}{2}\right)$

$x = -\frac{\overset{11}{\cancel{33}}}{\underset{8}{\cancel{16}}} \cdot \frac{\cancel{2}}{\underset{9}{\cancel{27}}} + \frac{3}{2} = -\frac{11}{72} + \frac{108}{72}$

$= \frac{97}{72}$

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3. Find the absolute extrema of $f(x) = 2x - 3x^{2/3}$ on [-1, 3]

$f'(x) = 2 - 2x^{-\frac{1}{3}}$

$= 2 - \frac{2}{\sqrt[3]{x}}$

$= \frac{2\sqrt[3]{x} - 2}{\sqrt[3]{x}}$

$x = 0, 1$

min $= -5$

max $= 0$

Test

$f(-1) = -5$

$f(0) = 0$

$f(1) = -1$

$f(3) = 6 - 3\sqrt[3]{9}$

4. A container in the shape of a right circular cylinder with no top has a surface area of 3π ft². What height, h, and radius, r, will maximize the volume of the cylinder?

$S = \cancel{\pi} r^2 + 2\cancel{\pi} rh = 3\cancel{\pi}$

$h = \frac{3 - r^2}{2r}$

$V = \pi r^2 h$

$V = \pi r^{\cancel{2}}\left(\frac{3 - r^2}{2\cancel{r}}\right)$

$= \frac{3}{2}\pi r - \frac{\pi}{2} r^3$

$\frac{dV}{dr} = \frac{3}{2}\pi - \frac{3}{2}\pi r^2 = 0$

$r = 1$ ft

$h = \frac{3-1}{2} = 1$ ft

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5. Find all values of c that satisfy the mean value theorem for derivatives for the function $y = 2x^3 - 3x$ on [-1, 2]

y is cont on $[-1,2]$ ✓
y is diff on $(-1,2)$ ✓

$sos = \frac{y(2)-y(-1)}{2-(-1)} = \frac{16-6-(-2+3)}{3} = 3$

$y' = 6x^2 - 3 = 3$
$x^2 = 1,\ x = \pm 1$

$c = 1$

6. Given the function $y = x^4 - 5x^2 - 3x + 1$, identify the x- and y-intercepts, the increasing and decreasing intervals, the relative extrema, the intervals of concavity, and the points of inflection. (You may need to use your calculator for the "solving" feature)

y-int = 1
x-int ≈ -1.699, -1, 0.239, 2.460

y': − a + b − c +
↘ a ↗ b ↘ c ↗

$y' = 4x^3 - 10x - 3 = 0$
$x \approx$ a = −1.401771, b = −0.3121682, c = 1.7139394

y is inc on $(a,b) \cup (c,\infty)$ b/c $y' > 0$ on those intervals

y is dec on $(-\infty,a) \cup (b,c)$ b/c $y' < 0$ on those intervals

rel max @ (b, 1.458) b/c y' chgs from + to − at b

rel min @ (a, −0.758), (c, −10.200) b/c y' chgs from − to + at a and c

$y'' = 12x^2 - 10 = 0$
$x = \pm\sqrt{\frac{5}{6}}$

y'': + $-\sqrt{\frac{5}{6}}$ − $\sqrt{\frac{5}{6}}$ +
∪ ∩ ∪

y is ∪ on $\left(-\infty, -\sqrt{\frac{5}{6}}\right) \cup \left(\sqrt{\frac{5}{6}}, \infty\right)$ b/c $y'' > 0$ on those intervals

y is ∩ on $\left(-\sqrt{\frac{5}{6}}, \sqrt{\frac{5}{6}}\right)$ b/c $y'' < 0$ on that interval

poi occur @ $\left(-\sqrt{\frac{5}{6}}, -.912\right), \left(\sqrt{\frac{5}{6}}, -5.210\right)$
b/c y'' chgs sign @ those points

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