

Chapter 3 Review

1. Find the differential, dy , of $y = -2\sec^3(\sin(4x))$
2. Complete two iterations of Newton's Method to approximate $3^{1/4}$ starting with a value of 1
3. Find the absolute extrema of $f(x) = 2x - 3x^{2/3}$ on $[-1, 3]$
4. A container in the shape of a right circular cylinder with no top has a surface area of $3\pi \text{ ft}^2$. What height, h , and radius, r , will maximize the volume of the cylinder?
5. Find all values of c that satisfy the mean value theorem for derivatives for the function $y = 2x^3 - 3x$ on $[-1, 2]$
6. Given the function $y = x^4 - 5x^2 - 3x + 1$, identify the x - and y -intercepts, the increasing and decreasing intervals, the relative extrema, the intervals of concavity, and the points of inflection. (You may need to use your calculator for the "solving" feature)

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1. Find the differential, dy , of $y = -2\sec^3(\sin(4x))$

$$\begin{aligned} dy &= -2(3)\sec^2(\sin(4x))\sec(\sin(4x))\tan(\sin(4x))\cos(4x)(4)dx \\ &= -24\sec^3(\sin(4x))\tan(\sin(4x))\cos(4x)dx \end{aligned}$$

2. Complete two iterations of Newton's Method to approximate $3^{1/4}$ starting with a value of 1

$$\begin{array}{lll} x = \sqrt[4]{3} & y(1) = -2 & y\left(\frac{3}{2}\right) = \frac{81}{16} - \frac{48}{16} = \frac{33}{16} \\ x^4 = 3 & y'(1) = 4 & y'\left(\frac{3}{2}\right) = 4\left(\frac{27}{8}\right) = \frac{27}{2} \\ y = x^4 - 3 & y + 2 = 4(x - 1) & y - \frac{33}{16} = \frac{27}{2}\left(x - \frac{3}{2}\right) \\ y' = 4x^3 & x_2 = \frac{1}{2} + 1 = \frac{3}{2} & x = -\frac{\cancel{33}}{8} \cdot \frac{\cancel{2}}{\cancel{27}} + \frac{3}{2} = -\frac{11}{72} + \frac{108}{72} \\ & & = \frac{97}{72} \end{array}$$

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3. Find the absolute extrema of $f(x) = 2x - 3x^{2/3}$ on $[-1, 3]$

$$f'(x) = 2 - 2x^{-1/3} \quad x=0, 1$$

$$= 2 - \frac{2}{\sqrt[3]{x}}$$

$$= \frac{2\sqrt[3]{x} - 2}{\sqrt[3]{x}}$$

$$\text{min} = -5$$

$$\text{max} = 0$$

Test

$$f(-1) = -5$$

$$f(0) = 0$$

$$f(1) = -1$$

$$f(3) = 6 - 3\sqrt[3]{9}$$

4. A container in the shape of a right circular cylinder with no top has a surface area of $3\pi \text{ ft}^2$. What height, h , and radius, r , will maximize the volume of the cylinder?

$$S = \pi r^2 + 2\pi r h = 3\pi$$

$$h = \frac{3 - r^2}{2r}$$

$$V = \pi r^2 h$$

$$V = \pi r^2 \left(\frac{3 - r^2}{2r} \right)$$

$$= \frac{3}{2}\pi r - \frac{\pi}{2}r^3$$

$$\frac{dV}{dr} = \frac{3}{2}\pi - \frac{3}{2}\pi r^2 = 0$$

$$r = 1 \text{ ft}$$

$$h = \frac{3 - 1}{2} = 1 \text{ ft}$$

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5. Find all values of c that satisfy the mean value theorem for derivatives for the function $y = 2x^3 - 3x$ on $[-1, 2]$

y is cont on $[-1, 2]$ ✓
 y is diff on $(-1, 2)$ ✓

$$y' = 6x^2 - 3 = 3$$

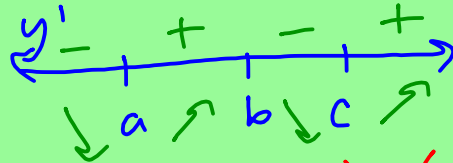
$$x^2 = 1, x = \pm 1$$

$$\text{sos} = \frac{y(2) - y(-1)}{2 - (-1)} = \frac{16 - 6 - (-2 + 3)}{3} = 3 \quad c = 1$$

6. Given the function $y = x^4 - 5x^2 - 3x + 1$, identify the x - and y -intercepts, the increasing and decreasing intervals, the relative extrema, the intervals of concavity, and the points of inflection. (You may need to use your calculator for the "solving" feature)

$$y\text{-int} = 1$$

$$x\text{-int} \approx -1.699, -1.0739, 2.460$$



$$y' = 4x^3 - 10x - 3 = 0$$

$$a = -1.401771,$$

$$x \approx b = -0.3121682$$

$$c = 1.7139394$$

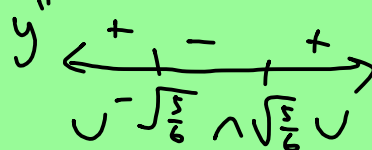
y is inc on $(a, b) \cup (c, \infty)$
 b/c $y' > 0$ on those intervals

y is dec on $(-\infty, a) \cup (b, c)$
 b/c $y' < 0$ on those intervals

rel max @ $(b, 1.458)$ b/c y' chgs from + to - at b
 rel min @ $(a, -0.758), (c, -10.200)$
 b/c y' chgs from - to + at a and c

$$y'' = 12x^2 - 10 = 0$$

$$x = \pm \sqrt{\frac{5}{6}}$$



y is U on $(-\infty, -\sqrt{\frac{5}{6}}) \cup (\sqrt{\frac{5}{6}}, \infty)$ b/c $y'' > 0$
 on those intervals

y is N on $(-\sqrt{\frac{5}{6}}, \sqrt{\frac{5}{6}})$ b/c $y'' < 0$ on that interval

poi occur @ $(-\sqrt{\frac{5}{6}}, -9.12), (\sqrt{\frac{5}{6}}, -5.210)$
 b/c y'' chgs sign @ those points

