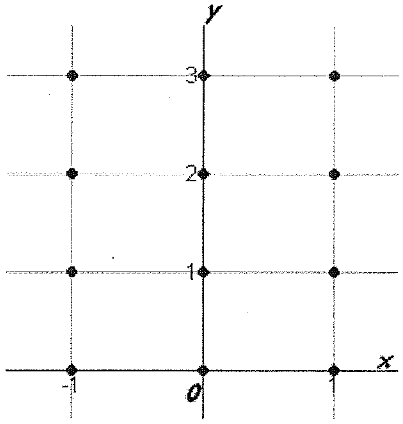
AP Calculus AB **Morning Test Review : Ch. 5, 6, Slope Fields**

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$

a. Sketch a slope field through the indicated points



b. Find the particular solution y = f(x) to the differential equation with the initial condition f(1) = 1

c. Write the equation for the tangent line to the curve y = f(x) through the point (1,1)

d. Use the tangent line equation to estimate the value of f(1.2)

2. Newton's Law of Cooling: A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F. The initial temperature of the liquid is 160 °F. After 5 minutes, the liquid's temperature is 60 °F. How long will it take for its temperature to decrease to 30 °F?

Use the differential equation $\frac{dy}{dt} = k(y - 20)$

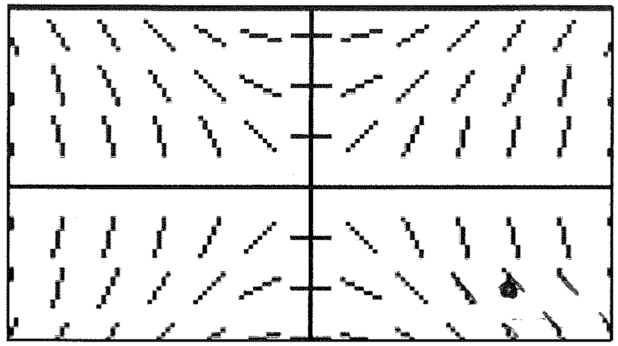
3. Match the slope field with the correct equation:

a) $\frac{dy}{dx} = \frac{-x}{y}$

b) $\frac{dy}{dx} = \frac{-y}{x}$

c) $\frac{dy}{dx} = \frac{x}{y}$

d) $\frac{dy}{dx} = \frac{-1}{x-y}$



3b. Find the particular solution given y(5) = -2

3c. Sketch the solution through the given point and find the domain

4. $\int \frac{2e^{x}-2e^{-x}}{(e^{x}+e^{-x})^2}\, dx$

5. $\int (-sinx)(3^{cosx})dx$

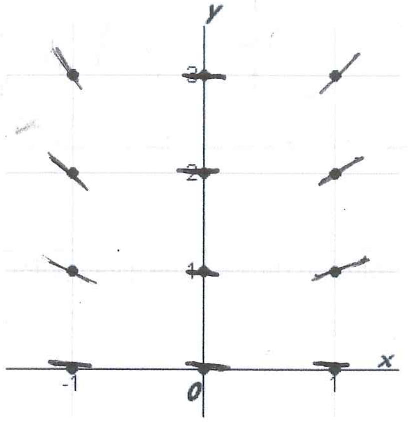
6. $\int \frac{8}{\sqrt{4-x^2+2x}}\, dx$ (hint: complete the square)

7. $\int \csc(1-3x)\, dx$

AP Calculus AB **Morning Test Review : Ch. 5, 6, Slope Fields** Solution Key

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$

a. Sketch a slope field through the indicated points



b. Find the particular solution y = f(x) to the differential equation with the initial condition f(1) = 1

$\int \frac{dy}{y} = \int \frac{1}{2} x\,dx$

$\ln|y| = \frac{1}{2}\left(\frac{x^2}{2}\right) + c$

$e \qquad e$

$y = e^{x^2/4} \cdot e^{c}$

$y = Ce^{x^2/4}$

$1 = Ce^{1/4}$

$\frac{1}{e^{1/4}} = C$

$C = e^{-1/4}$

$y = e^{-1/4} \cdot e^{x^2/4}$

$\boxed{y = e^{\frac{x^2-1}{4}}}$

c. Write the equation for the tangent line to the curve y = f(x) through the point (1,1)

$\frac{dy}{dx}\Big|_{(1,1)} = \frac{(1)(1)}{2} = \frac{1}{2}$

point: $(1,1)$

$m = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x-1)$

$\boxed{y = \frac{1}{2}(x-1) + 1}$

d. Use the tangent line equation to estimate the value of f(1.2)

$f(1.2) \approx \frac{1}{2}(1.2-1) + 1 = 1.1$

$\boxed{f(1.2) \approx 1.1}$

2. Newton's Law of Cooling: A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F. The initial temperature of the liquid is 160 °F. After 5 minutes, the liquid's temperature is 60 °F. How long will it take for its temperature to decrease to 30 °F?

Use the differential equation $\frac{dy}{dt} = k(y - 20)$

(time, temperature)

$(0, 160)$

$(5, 60)$

$(\_, 30)$

$\int \frac{dy}{y-20} = \int k\,dt$

$\ln|y-20| = kt + c$

$e^{\ln|y-20|} = e^{kt} \cdot e^{c}$

$\boxed{y - 20 = Ce^{kt}}$

$160 - 20 = Ce^{k(0)}$

$140 = C(1)$

$140 = C$

$y - 20 = 140e^{kt}$

$60 - 20 = 140e^{k(5)}$

$40 = 140e^{5k}$

$\frac{40}{140} = e^{5k}$

$\frac{2}{7} = e^{5k}$

$\ln(2/7) = \ln e^{5k}$

$\ln(2/7) = 5k$

$\frac{1}{5}\ln(2/7) = k$

$y - 20 = 140e^{\frac{1}{5}\ln(2/7)t}$

$30 - 20 = 140e^{\frac{1}{5}\ln(2/7)t}$

$10 = 140e^{\frac{1}{5}\ln(2/7)t}$

$\frac{1}{14} = e^{\frac{1}{5}\ln(2/7)t}$

$\ln(1/14) = \ln e^{\frac{1}{5}\ln(2/7)t}$

$\ln(1/14) = \frac{1}{5}\ln(2/7)t$

$\frac{5\ln(1/14)}{\ln(2/7)} = t$

$\boxed{t \approx 10.533 \text{ mins.}}$

3. Match the slope field with the correct equation:

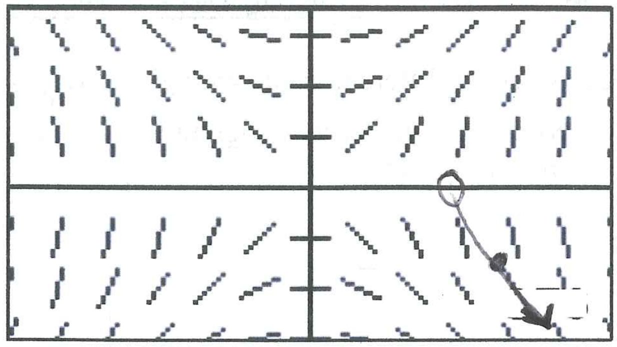
a) $\frac{dy}{dx} = \frac{-x}{y}$

b) $\frac{dy}{dx} = \frac{-y}{x}$

c) $\boxed{\frac{dy}{dx} = \frac{x}{y}}$

when $y=0$ slope undefined
when $x=0$ slope $=0$
at $(1,1)$ slope is positive

d) $\frac{dy}{dx} = \frac{-1}{x-y}$



3b. Find the particular solution given y(5) = -2

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$

$y^2 = x^2 + c$

$(-2)^2 = (5)^2 + c$

$\boxed{-21 = C}$

$y^2 = x^2 - 21$

$y = \pm\sqrt{x^2-21}$

$\boxed{y = -\sqrt{x^2-21}}$

3c. Sketch the solution through the given point and find the domain * the graph interval starts at $y=0$

$0 = -\sqrt{x^2-21}$

$(0)^2 = (\sqrt{x^2-21})^2$

$0 = x^2 - 21$

$21 = x^2 \quad x = \pm\sqrt{21}$

$\boxed{\text{Domain: } (\sqrt{21}, \infty)}$

4. $\int \frac{2e^x - 2e^{-x}}{(e^x+e^{-x})^2}\,dx$

$u = e^x + e^{-x}$

$\frac{du}{dx} = e^x(1) + e^{-x}(-1)$

$\frac{du}{dx} = e^x - e^{-x} \quad dx = \frac{du}{e^x - e^{-x}}$

$\int \frac{2\cancel{(e^x - e^{-x})}}{u^2} \cdot \frac{du}{\cancel{e^x - e^{-x}}}$

$= 2\int \frac{1}{u^2}\,du$

$= 2\int u^{-2}\,du$

$2\left(\frac{u^{-1}}{-1}\right) + c$

$= \frac{-2}{u} + c = \boxed{\frac{-2}{e^x + e^{-x}} + C}$

5. $\int(-\sin x)(3^{\cos x})dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$\int \cancel{-\sin x} \cdot 3^u \cdot \frac{du}{\cancel{-\sin x}}$

$\int 3^u\,du = \frac{3^u}{\ln 3} + c$

$= \boxed{\frac{3^{\cos x}}{\ln 3} + C}$

6. $\int \frac{8}{\sqrt{4-x^2+2x}}\,dx$ (hint: complete the square)

$\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$

$4 - (x^2 - 2x + \_) + \_$

$4 - (x^2 - 2x + 1) + 1$

$3 - (x-1)^2$

$\int \frac{8}{\sqrt{3-(x-1)^2}}\,dx$

$\int \frac{8}{\sqrt{(\sqrt{3})^2 - (x-1)^2}}\,dx$

$u = x - 1$

$\frac{du}{dx} = 1$

$a = \sqrt{3}$

$\int \frac{8}{\sqrt{a^2-u^2}}\,du$

$8\int \frac{du}{\sqrt{a^2-u^2}}$

$8 \cdot \arcsin\left(\frac{u}{a}\right) + C$

$= \boxed{8\arcsin\left(\frac{x-1}{\sqrt{3}}\right) + C}$

7. $\int \csc(1-3x)\,dx$

$u = 1 - 3x$

$\frac{du}{dx} = -3$

$dx = \frac{du}{-3}$

$\int \csc u \cdot \frac{du}{-3} = \frac{-1}{3}\int \csc u\,du$

$= \frac{-1}{3} \cdot -\ln|\csc u + \cot u| + C$

$= \boxed{\frac{1}{3}\ln|\csc(1-3x) + \cot(1-3x)| + C}$