AP CALCULUS AB **Ch. 7 Test review WS #2** **Area, Volume, and IBP/Tab**

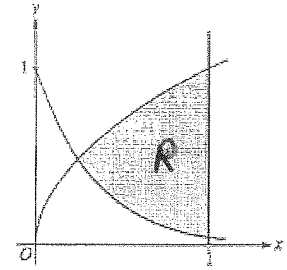
1. 2003 AB 1 and BC 1

   Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.



2. Consider the area enclosed by $x = \sqrt{y}$, x = 2, and y = 0. Let y = k be the line that divides the area into two equal parts. a) Find area of enclosed region b) Find k.

3. Let R be the region in the first quadrant bounded by the graph of $y = 4 - x^{2/3}$, the x-axis, and the y-axis.

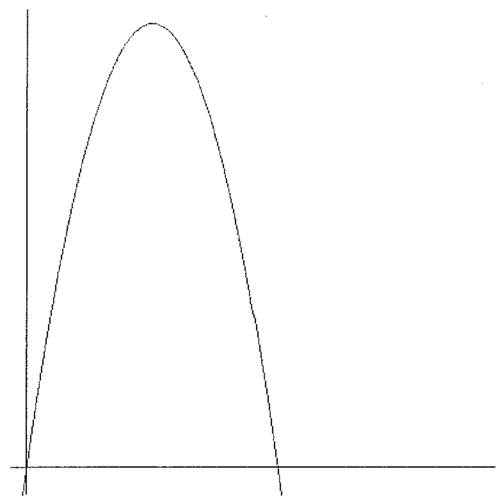
   a. Find the area of region R

   b. Find the volume of the solid generated when R is rotated around the x-axis

   c. The vertical line x = k divides the region R into two regions so that when these regions are rotated around the x-axis, they generate solids with equal volumes. Find the value of k.

4. Consider the graph $f(x) = -\frac{1}{2}x^2 + 8x$. A line is drawn tangent to the curve at x = 10.
   a. Find the equation of the tangent line in point-slope form and slope-intercept form

   b. Find the area of the region bounded by the graph f(x), the tangent line, and the x-axis



5. The base of a solid is the region enclosed by the curve $x = 2\sqrt{y}$ and the lines $x + y = 0$ and $y = 4$. Find the volume of the solid if all cross sections perpendicular to the y-axis are right isosceles triangles having the hypotenuse with one endpoint on the line $x + y = 0$ and the other on the curve $x = 2\sqrt{y}$.

<u>Evalute the following integrals:</u>

6. $\int \ln 3x dx$

7. $\int \frac{4x^3}{e^{-3x}} dx$

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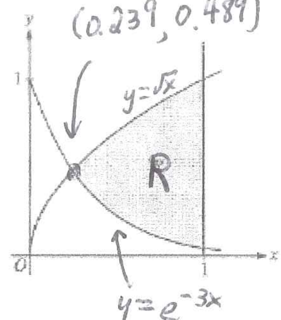
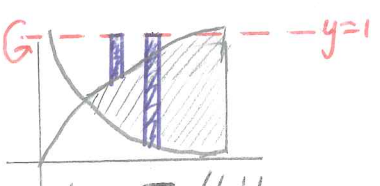


a) $\int_{0.239}^{1} \underbrace{\sqrt{x}}_{\text{top}} - \underbrace{e^{-3x}}_{\text{bottom}}\,dx = \boxed{0.442 \text{ units}^2}$

b)



Washer, Top/bottom

$R(x)=1-e^{-3x}$

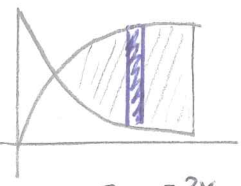
$r(x)=1-\sqrt{x}$

$V=\pi\int_{0.239}^{1} R(x)^2 - r(x)^2\,dx$

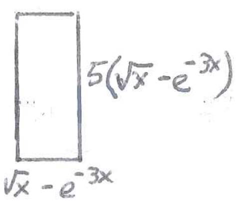
$V=\int_{0.239}^{1} [1-e^{-3x}]^2 - [1-\sqrt{x}]^2\,dx$

$=\boxed{0.453\pi \text{ units}^3}$

c)



base $=\sqrt{x}-e^{-3x}$



Area = base × height

$V=\int [\text{Area cross section}]\,dx$

$V=\int_{0.239}^{1} [\sqrt{x}-e^{-3x}]\cdot 5(\sqrt{x}-e^{-3x})\,dx$

$=5\int_{0.239}^{1} (\sqrt{x}-e^{-3x})^2\,dx$

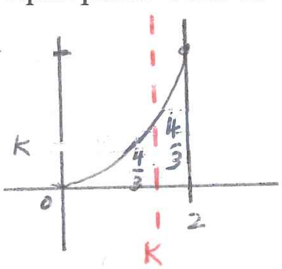
$=\boxed{1.554}$ units^3

2. Consider the area enclosed by $x=\sqrt{y}$, $x=2$, and $y=0$. Let $y=k$ be the line that divides the area into two equal parts. Find k.

$y=x^2$ (1st quadrant)

$\sqrt{y}=2$

$y=4$



a) $\int_0^2 x^2\,dx = \frac{x^3}{3}\Big]_0^2 = \frac{2^3}{3} = \frac{8}{3}$

b) $\int_0^k x^2\,dx = \frac{4}{3}$

$\frac{x^3}{3}\Big]_0^k = \frac{k^3}{3} - \frac{0^3}{3}$

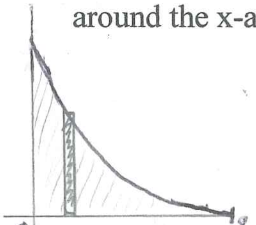
$\frac{k^3}{3} = \frac{4}{3}$ $k^3=4$ $\boxed{k=\sqrt[3]{4}\approx 1.587}$

3. Let R be the region in the first quadrant bounded by the graph of $y=4-x^{2/3}$, the x-axis, and the y-axis.

a. Find the area of region R

b. Find the volume of the solid generated when R is rotated around the x-axis

c. The vertical line x = k divides the region R into two regions so that when these regions are rotated around the x-axis, they generate solids with equal volumes. Find the value of k.



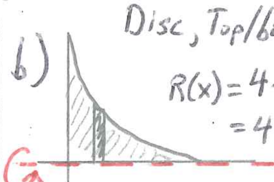
$4-x^{2/3}=0$ $x=4^{3/2}=8$

a) $A=\int_0^8 4-x^{2/3}\,dx$

$\boxed{A=12.799 \text{ units}^2}$

Disc, Top/bottom

b)



$R(x)=4-x^{2/3}-0$

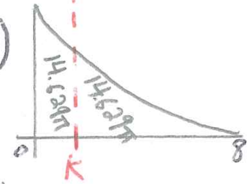
$=4-x^{2/3}$

$V=\pi\int_0^8 R(x)^2\,dx$

$V=\pi\int_0^8 [4-x^{2/3}]^2\,dx$

$=\boxed{29.257\pi \text{ units}^3}$

c)



$\pi\int_0^k [4-x^{2/3}]^2\,dx = 14.629\pi$

$\int 16-8x^{2/3}+x^{4/3}\,dx$

$16x - \frac{8x^{5/3}}{5/3} + \frac{x^{7/3}}{7/3}\Big]_0^k$

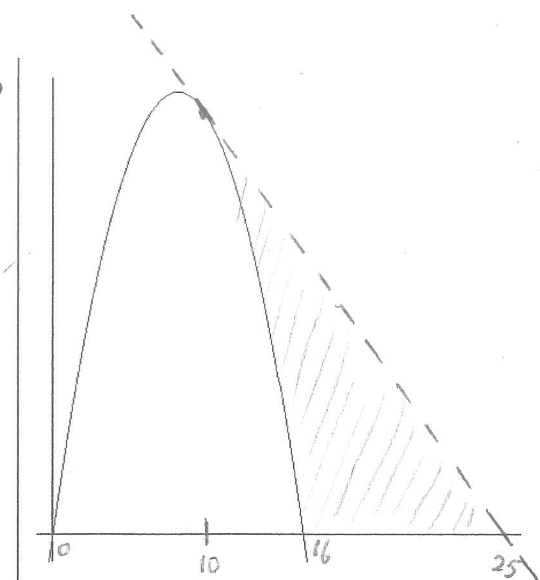
~~$\pi\,16k-\frac{3}{5}\cdot 8k^{5/3}+\frac{3}{7}k^{7/3}=14.629\pi$~~

* set equation equal to zero. Graph and look for x-int.

$16k-\frac{24}{5}k^{5/3}+\frac{3}{7}k^{7/3}-14.629=0$

$\boxed{K=1.361}$

4. Consider the graph $f(x) = -\frac{1}{2}x^2 + 8x$. A line is drawn tangent to the curve at x = 10.



a. Find the equation of the tangent line in point-slope form and slope-intercept form

point: $(10, \_)$
slope: $m = \_\_$
$y - y_1 = m(x - x_1)$

$f(10) = 30$
$f'(x) = \frac{-1}{2} \cdot 2x + 8$
$f'(x) = -x + 8$
$f'(10) = -10 + 8 = -2$

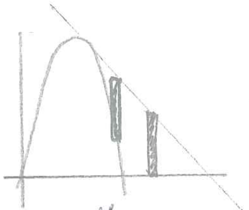
point: $(10, 30)$
$m = -2$
$y - y_1 = m(x - x_1)$
$\underline{y - 30 = -2(x - 10)}$

$y = -2(x-10) + 30$
$y = -2x + 20 + 30$
$\underline{y = -2x + 50}$

b. Find the area of the region bounded by the graph f(x), the tangent line, and the x-axis

* find bounds:
$-\frac{1}{2}x^2 + 8x = 0$
$-\frac{1}{2}x(x - 16) = 0$
$x = 0, 16$

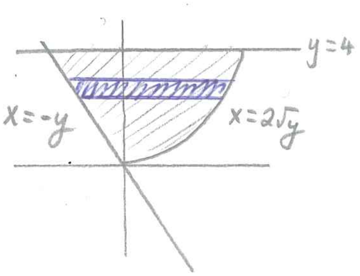
$-2x + 50 = 0$
$-2x = -50$
$x = 25$



$$A = \int_{10}^{16} \underbrace{-2x+50}_{top} - \underbrace{\left(\tfrac{-1}{2}x^2 + 8x\right)}_{bottom} dx + \int_{16}^{25} \underbrace{-2x+50}_{top} - \underbrace{0}_{bottom} dx$$

$$36 \quad + \quad 81 \quad = \boxed{117 \text{ units}^2}$$

5. The base of a solid is the region enclosed by the curve $x = 2\sqrt{y}$ $\rightarrow y = \frac{x^2}{4}$ and the lines $x + y = 0$ $\rightarrow y = -x$ and $y = 4$. Find the volume of the solid if all cross sections perpendicular to the y-axis are right isosceles triangles having the hypotenuse with one endpoint on the line $x + y = 0$ and the other on the curve $x = 2\sqrt{y}$.



base $= 2\sqrt{y} - (-y)$
$= 2\sqrt{y} + y$

$A = \frac{1}{4}(\text{hypotenuse})^2$
$A = \frac{1}{4}(2\sqrt{y} + y)^2$

$V = \int [\text{Area of cross section}] dx$
$= \int_0^4 \frac{1}{4}[2\sqrt{y} + y]^2 dy = \boxed{26.133 \text{ units}^3}$

Evalute the following integrals:

6. $\int \ln 3x dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln(3x)$ $dv = dx$

$\frac{du}{dx} = \frac{3}{3x} = \frac{1}{x}$ $v = x$

$du = \frac{1}{x}dx$

$x\ln(3x) - \int x\left(\frac{1}{x}\right)dx$

$\int 1\,dx = x$

$\boxed{x\ln(3x) - x + C}$

7. $\int \frac{4x^3}{e^{-3x}} dx = \int 4x^3 e^{3x} dx$

u	dv
$+\ 4x^3$	e^{3x}
$-\ 12x^2$	$\frac{1}{3}e^{3x}$
$+\ 24x$	$\frac{1}{9}e^{3x}$
$-\ 24$	$\frac{1}{27}e^{3x}$
$+\ 0$	$\frac{1}{81}e^{3x}$

$\int e^{3x} dx$
$u = 3x$ $\int e^u \cdot \frac{du}{3}$
$\frac{du}{dx} = 3$ $\frac{1}{3}\int e^u du$
$dx = \frac{du}{3}$
$= \frac{1}{3}e^{3x} + C$

$$\boxed{e^{3x}\left(\frac{4x^3}{3} - \frac{12x^2}{9} + \frac{24x}{27} - \frac{24}{81}\right) + C}$$