BC Final Exam Review
Multiple Choice Answers

NO CALCULATOR SECTION

1) B
2) A
3) A
4) B
5) C
6) D
7) A
8) C
9) A
10) D
11) B
12) B
13) A
14) A
15) B
16) D
17) B
18) C
19) B
20) C
21) D
22) A
23) B
24) B
25) C
26) A
27) B
28) B

CALCULATOR SECTION

76) C
77) A
78) B
79) D
80) A
81) B
82) A
83) A
84) C
85) D
86) B
87) B
88) A
89) D
90) C
91) D
92) D

BC Final Exam Review
Free Response Solutions

1)

a) $4\ln x = \frac{12}{x+1}$ $at\ \ x \approx 2.410$

$$\int_1^{2.410} \left(\frac{12}{x+1} - 4\ln x\right) dx \approx 3.563$$

1 – limits
1 – integrand
1 – answer

b)
WASHER METHOD

$$\pi \int_0^{3.519} \left(\left(e^{\frac{y}{4}} - 1\right)^2\right) dy + \pi \int_{3.519}^{6} \left(\frac{12}{y} - 1\right)^2 dy$$

≈ 9.702 or 9.703

SHELL METHOD

$$2\pi \int_1^{2.410} (x-1)\left(\frac{12}{x+1} - 4\ln x\right) dx$$

$\approx 9.702\ or\ 9.703$

1 – limits and constant
1 – integrand
1 – answer

c) length of one side =

$$\frac{12}{x+1} - 4\ln x$$

$$V = \int_1^{2.410} \frac{\sqrt{3}}{4}\left(\frac{12}{x+1} - 4\ln x\right)^2 dx$$

$\approx 5.661\ or\ 5.662$

1 – limits
1 – integrand
1 – answer

2)

a) $\frac{dy}{dx} = \frac{2t}{3t^2 - 4}$

$\left.\frac{dy}{dx}\right|_{t=3} = \frac{6}{23}$

1 – dy/dx
1 – answer

b) $x(5) = \int_1^5 \frac{dx}{dt}\,dt - 3 = 105$

1 – integral
1 – uses initial condition
1 – answer

c) $speed = \sqrt{(3t^2 - 4)^2 + (2t)^2}$

$speed|_{t=5} = \sqrt{5141} \approx 71.700\ or\ 71.701$

1 – speed in terms of t
1 – answer

d) $a(t) = \langle \left(\frac{dx}{dt}\right)', \left(\frac{dy}{dt}\right)' \rangle = \langle 6t, 2 \rangle$

$a(5) = \langle 30,2 \rangle$

1 – acceleration in terms of t
1 – a(5)

3)

a) $\frac{r}{h} = \frac{4}{16} \quad so \quad r = \frac{1}{4}h$

$$V = \frac{\pi}{48}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{16}h^2\frac{dh}{dt} = 2\pi h^3$$

$$\frac{dh}{dt} = 32h$$

1 – volume in terms of h
1 – $\frac{dV}{dt}$
1 - answer

b) $\frac{dh}{dt} = 32h$

$$\int \frac{1}{h}\,dh = \int 32\,dt$$

$$\ln h = 32t + C$$

$$h = Ae^{32t}$$

$$12 = Ae^0 \;\; so \;\; A = 12$$

$$h = 12e^{32t}$$

1 – separation of variables
1 – antiderivatives
1 – constant of integration
1 – uses initial condition
1 – answer

c) $12e^{32t} = 16$

$$e^{32t} = \frac{4}{3}$$

$$t = \frac{1}{32}\ln\frac{4}{3}$$

1 - answer

4)

$a)\ \lim_{n\to\infty}\left|\frac{x^{2n+2}}{2n+3}\cdot\frac{2n+1}{x^{2n}}\right| = |x^2| < 1$

$\sum_{n=0}^{\infty}(-1)^n\frac{(-1)^{2n}}{2n+1}$ converges by alternating series test

$\sum_{n=0}^{\infty}(-1)^n\frac{(1)^{2n}}{2n+1}$ converges by alternating series test

interval of convergence: $-1 \le x \le 1$

1 – ratio test setup
1 – evaluation of limit
1 – test endpoints
1 – answer

$b)\ f\left(\frac{1}{3}\right) \approx 1 - \frac{1}{27} + \frac{1}{405} = \frac{391}{405}$

1 – answer

$c)\ f'(x) = -\frac{2x}{3} + \frac{4x^3}{5} - \frac{6x^5}{7} + \frac{8x^7}{9} - \cdots + \frac{(-1)^n(2n)x^{2n-1}}{2n+1} + \cdots$

2 – four terms
1 – general term

5)

a) $g(2) = \int_0^2 f(t)dt = -\frac{1}{2}(2)(2) = -2$

$g(-2) = \int_0^{-2} f(t)dt = -\int_{-2}^{0} f(t)dt = -\pi$

1 – g(2)
1 – g(-2)

b) $g'(x) = f(x)$
$g'(1) = f(1) = -2$

1 – relates g to f
1 – answer

c) Points of inflection occur at $x = -2, 1$ because $g''(x)$ changes sign at these values

($g'(x)$ changes from increasing to decreasing at *x* = -2 and decreasing to increasing at *x* = 1)

1 – answer
1 – justification

d) $g'(x) = 0 \ at \ x = -4, 0, 2, 4$

$g(-4) = \int_0^{-4} f(t)dt = -\int_{-4}^{0} f(t)dt = -2\pi$
$g(0) = 0$
$g(2) = -2$
$g(4) = -\frac{1}{4}$

Absolute minimum occurs at *x* = -4

1 – identifies candidates
1 – tests values
1 – answer

6)

a)

$A(0) = 1200$

$\left.\frac{dA}{dt}\right|_{t=0} = \frac{3}{10}(1200 - 40) = 348$

$y - 1200 = 348(t - 0)$ *or*

$y = 348t + 1200$

$y(5) = 348(5) + 1200 = 2940$

so $A(5) \approx 2940$

1 – dA/dt
1 – tangent line equation
1 – approximation

b)

$$\frac{d^2A}{dt^2} = \frac{3}{10}\frac{dA}{dt} = \frac{3}{10}\left(\frac{3}{10}(A - 40)\right)$$

$$= \frac{9}{100}(A - 40)$$

1 – answer

c)

$$\frac{1}{A - 40}dA = \frac{3}{10}dt$$

$$\ln|A - 40| = \frac{3}{10}t + C$$

$$A - 40 = Me^{\frac{3}{10}t}$$

$$A = Me^{\frac{3}{10}t} + 40$$

$A(0) = M + 40 = 1200$ *so* $M = 1160$

$$A = 1160e^{\frac{3}{10}t} + 40$$

1 – separation of variables
1 – antiderivatives
1 – constant
1 – uses initial condition
1 – answer