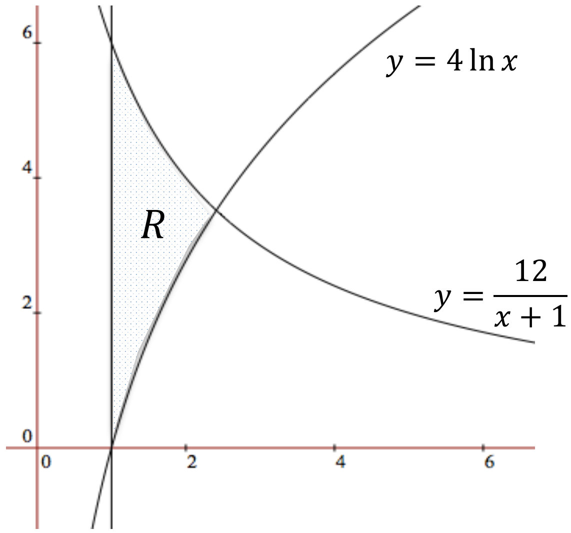
AP CALCULUS BC FINAL EXAM REVIEW **NAME:** ________________________________

CALCULATOR PORTION – 30 minutes



1. Let R be the region bounded by the graphs of $y = 4\ln x$, $y = \frac{12}{x+1}$, and $x = 1$ as shown in the figure above.

a) Find the area of R.
b) Find the volume of the solid that results when R is revolved about the vertical line $x = 1$.
c) Let R be the base of a solid. Every cross section of the solid perpendicular to the x-axis is an equilateral triangle with one side across the base. Find the volume of the solid.

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2. For $t \geq 0$, a particle is moving along a curve with its position at time t given by $(x(t), y(t))$ and $\frac{dx}{dt} = 3t^2 - 4$, $\frac{dy}{dt} = 2t$. At time $t = 1$, its position is (-3, 1).

a) Find the slope of the particle's path at time $t = 3$.
b) Find the x-coordinate of the particle's position at time $t = 5$.
c) Find the speed of the particle at time $t = 5$.
d) Find the acceleration vector of the particle at time $t = 5$.

AP CALCULUS AB FINAL EXAM REVIEW

NO CALCULATOR PORTION – 60 minutes

3. A water storage tank in the shape of an inverted cone whose height is 16 meters and radius is 4 meters. The tank initially contains 36π m^3. The water is filling from the tank at a rate of $2\pi h^3$ m^3/min. (Note: the volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

a) Find $\frac{dh}{dt}$ in terms of h.
b) If h = 12 m at time t = 0, find an equation for h as a function of t.
c) At what time is the storage tank full?

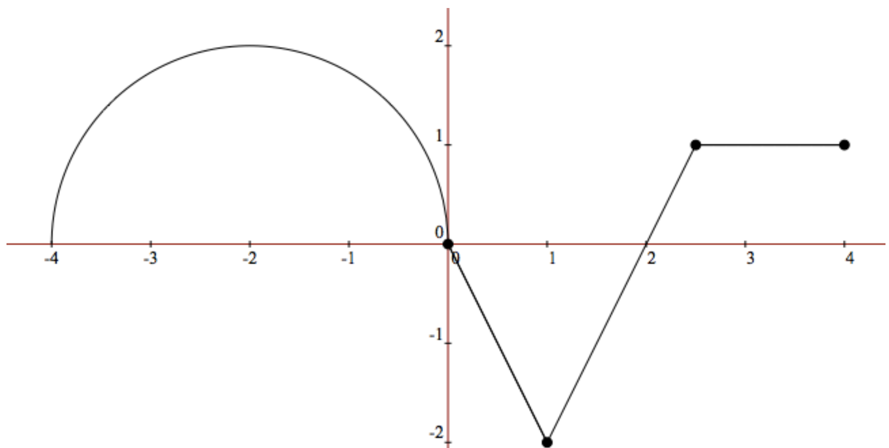
4. The function *f* is differentiable everywhere and the MacLaurin series is

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n+1} = 1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \cdots$$

(a) Use the ratio test to find the interval of convergence of the MacLaurin series of *f*.

(b) Approximate $f\left(\frac{1}{3}\right)$ using the first three nonzero terms.

(c) Write the first four nonzero terms and the general term for the MacLaurin series for $f'(x)$.



5. The figure above shows the graph of *f*, which is a continuous function defined on -4 ≤ *x* ≤ 4. The graph of *f* from *x* = -4 to *x* = 0 is a semi-circle, and from *x* = 0 to *x* = 4 consists of three line segments. Let $g(x) = \int_0^x f(t)dt$.

a) Find $g(2)$ and $g(-2)$.

b) Evaluate $g'(1)$.

c) Find all values of *x* where $g(x)$ has a point of inflection.

d) At what value of *x* is $g(x)$ a minimum?

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6. Bacteria is growing in a Petri dish at a rate that satisfies the differential equation $\frac{dA}{dt} = \frac{3}{10}(A - 40)$, where *A* is the number of bacteria and *t* is measured in days. There are initially 1200 bacteria in the dish.

a) Use the line tangent to the graph of *A* at *t* = 0 to estimate the number of bacteria the end of *t* = 5 days.

b) Find $\frac{d^2A}{dt^2}$ in terms of *A*.

c) Find the particular solution to the above differential equation with the initial condition $A(0) = 1200$.