Warmup!!

Consider the following sequences:

A) $a_n = \frac{2^n}{n^3}$ B) $a_n = \frac{2n+3}{n+n^2}$

$b_n = \frac{2n}{n^2} = \frac{2}{n}$

Find the following for each:

1) a_4

2) S_4 where $S_n = \sum_{n=1}^{\infty} a_n$

3) Does a_n converge or diverge?

4) Does $\sum_{n=1}^{\infty} a_n$ converge or diverge?

5) Does $\sum_{n=1}^{\infty} (-1)^n a_n$ converge or diverge?

6) Find n so S_n in the alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ is within 0.1 of S_∞, if $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

A)

1) $a_4 = \frac{16}{64} = \frac{1}{4}$ ✓

2) $S_4 = \frac{329}{108} \approx 3.046$

3) $\lim_{n\to\infty} a_n = \infty \Rightarrow$ sequence diverges

4) series diverges based on nth term test

5) alternating series also diverges based on nth term test

6) N/A

B)

1) $a_4 = \frac{11}{20} = 0.55$

✓ 2) $S_4 = \frac{149}{30} \approx 4.967$

3) $\lim_{n\to\infty} a_n = 0 \Rightarrow$ sequence converges

4) I chose to use the limit comparison test letting $b_n = \frac{2n}{n^2}$

b_n diverges based on the p-test

$$\lim_{n\to\infty} \frac{\frac{2n+3}{n+n^2}}{\frac{2n}{n^2}} = \lim_{n\to\infty} \frac{2n^3+3n^2}{2n^2+2n^3} = 1 > 0$$

so the series diverges based on the limit comparison test

5) $a_n > 0$

$$a_{n+1} = \frac{2(n+1)+3}{n+1+(n+1)^2} = \frac{2n+5}{n^2+3n+2} < \frac{2n+3}{n+n^2} = a_n$$

(I solved the inequality to verify)

$\boxed{\lim_{n\to\infty} a_n = 0}$ ✓

alternating series converges based on the alternating series test

6) $a_{21} < 0.1$, so S_{20} would be within 0.1 of S_∞

9-6b: It's time to play 'Name that Test!'

At the end of this lesson you will be able to:

- Look at a series and determine which test would be the most efficient and appropriate to use to determine convergence/divergence

NOTE: There is a homework change today!

March 08, 2016

WHICH TEST TO USE???

Start with the obvious

If the series:	Use the:
is not approaching 0	nth term test
is geometric - $a(r)^n$	geometric series test
is of the form $1/n^p$	p-test
is of the form - 'series - series'	or if bottom factors telescoping
is raised to nth power	root test
contains a factorial	ratio test
contains -1 to a power	alternating series test

Why does $\sum_{n=1}^{\infty} \frac{1}{n}$ diverge?

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \ldots$$

$$= 1 + \frac{1}{2} + \underbrace{\frac{1}{4} + \frac{1}{4}} + \underbrace{\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}}$$

$$= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots$$

$$= \infty$$

Trickier:

- If the series is easy to integrate, use the integral test
- If the series is very similar to another series (maybe just adding or subtracting a constant), use the direct or limit comparison tests
  - If you can easily determine which series is 'larger', use the direct comparison test
  - If you have trouble figuring out which series is 'larger' use the limit comparison test
- always try the ratio test!

Matching!! - Match each series with an appropriate test

1) $\sum_{n=1}^{\infty} \frac{1}{3n+1}$ F (E fails)

2) $\sum_{n=1}^{\infty} \frac{3(-1)^n}{4n+1}$ G

3) $\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$ A

4) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$ H

5) $\sum_{n=1}^{\infty} \left(\frac{\pi}{6}\right)^n$ B

6) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^n$ I

7) $\sum_{n=1}^{\infty} n \cdot e^{-n^2}$ D

a) nth term test

b) geometric series test

c) p-test

d) integral test

e) direct comparison test

f) limit comparison test

g) alternating series test

h) ratio test

i) root test

Now that you know which test to use, determine convergence/divergence for all of the series on the previous page.

1) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n+1}$ Since 1/(3n+1) < 1/(3n), you want to use a direct comparison, but because 1/(3n) diverges based on the p-test, the direct comparison test is inconclusive.

$$\lim_{n\to\infty} \frac{\frac{1}{3n+1}}{\frac{1}{3n}} = \lim_{n\to\infty} \frac{3n}{3n+1} = 1 > 0$$

So the series diverges based on the limit comparison test

2) $\displaystyle\sum_{n=1}^{\infty} \frac{3(-1)^n}{4n+1}$

$$\frac{3}{4n+1} > 0 \ \checkmark$$

$$\lim_{n\to\infty} \frac{3}{4n+1} = 0 \ \checkmark$$

$$\frac{3}{4(n+1)+1} = \frac{3}{4n+5} < \frac{3}{4n+1} \ \checkmark$$

So the series converges based on the alternating series test

3) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$

$$\lim_{n\to\infty} \frac{n+1}{3n+1} = \frac{1}{3} \neq 0$$

So the series diverges based on the nth term test

4) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)!}{10^{n+1}}}{\frac{n!}{10^n}} = \lim_{n\to\infty} \left(\frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!}\right) = \lim_{n\to\infty} \frac{n+1}{10} = \infty$$

So the series diverges based on the ratio test

5) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{6}\right)^n$ This series is geometric and converges because $|\pi/6| < 1$

6) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^n$

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n+1}{2n+1}\right)^n} = \lim_{n\to\infty} \frac{n+1}{2n+1} = \frac{1}{2} < 1$$

So the series converges based on the root test.

7) $\displaystyle\sum_{n=1}^{\infty} n \cdot e^{-n^2}$

$$\int_1^{\infty} xe^{-x^2}\,dx = \lim_{b\to\infty} \int_1^{b} xe^{-x^2}\,dx = \lim_{b\to\infty} \int_{-1}^{-b^2} -\frac{1}{2}e^u du$$

$$= \lim_{b\to\infty} -\frac{1}{2}e^u \Big|_{-1}^{-b^2} = \underline{\lim_{b\to\infty} \left(-\frac{1}{2}e^{-b^2} - -\frac{1}{2}e^{-1}\right)} = 0 + \underline{\frac{1}{2e}}$$

So the series converges based on the integral test

8) $\sum \frac{1}{n^2}$ $2 > 1$ conv

$$\sum_{n=1}^{\infty} ne^{-n^2} \text{ ? using root test}$$

$$\lim_{n\to\infty} \sqrt[n]{ne^{-n^2}}$$

$$= \lim_{n\to\infty} \sqrt[n]{n} \cdot \frac{1}{e^n} = \frac{1}{\infty} = 0$$

Let's take a shoot at a fun-filled Kahoot!

What have we learned?

- Can I look at a series and make an appropriate decision of which test to use to determine convergence/divergence?