March 01, 2017

Let's Warm Up

83. If the function f is continuous at $x = 3$, which of the following must be true?

(A) $f(3) < \lim_{x \to 3} f(x)$

(B) $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

(C) $f(3) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

(D) The derivative of f at $x = 3$ exists.

(E) The derivative of f is positive for $x < 3$ and negative for $x > 3$.

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

85. The fuel consumption of a car, in miles per gallon (mpg), is modeled by $F(s) = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}$, where s is the speed of the car, in miles per hour. If the car is traveling at 50 miles per hour and its speed is changing at the rate of 20 miles/hour^2, what is the rate at which its fuel consumption is changing?

(A) 0.215 mpg per hour

(B) 4.299 mpg per hour

(C) 19.793 mpg per hour

(D) 25.793 mpg per hour

(E) 515.855 mpg per hour

Warmup!!

1) Rewrite the following with no factorials

$$\frac{(2k-2)!}{(2k)!} = \frac{(2k-2)!}{(2k)(2k-1)(2k-2)!} = \frac{1}{(2k)(2k-1)}$$

2) Simplify completely (assume n > 1)

$$\frac{6^{n+2}}{2^n \cdot 3^{n-1}} = \frac{6^n \cdot 6^2}{2^n \cdot 3^n \cdot 3^{-1}} = \frac{36}{\frac{1}{3}} = 108$$

$$\frac{2^{n+2} \cdot 3^{n+2}}{2^n \cdot 3^{n-1}} = 2^2 \cdot 3^3 = 108$$

$\left(\frac{1}{2}\right)^n$

540

HW
9-5 #21

$$\sum_{n=1}^{\infty} \sin \frac{(2n-1)\pi}{2}$$

$$\lim_{n\to\infty} \sin \frac{(2n-1)\pi}{2}$$

$$\neq 0$$

diverges by nth term test

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$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$$

error: $\frac{1}{n} < .001$

$$\frac{1}{.001} < n$$

$$n > 1000$$

$$\sum_{n=1}^{1000} \frac{(-1)^{n+1}}{n} \approx .693$$

9-6a: Ratio and Root Tests!

Essential Learning Target

- In addition to examining the limit of the sequence of partial sums of a series, I can use a variety of methods for determining convergence or divergence including the nth term test, comparison test, limit comparison test, integral test, ratio test and alternating series test

Quick example: Take a look at the terms of a series. Determine if the series converges/diverges and state why:

63, 21, 7, 7/3, 7/9, ...

This series converges because it is geometric with a common ratio of 1/3 which is less than 1

Now take a look at the series below. Discuss with your group if the series converges or diverges and come up with a justification as to why. (Hint, you don't know a test for this yet but the last problem was a lead-in.)

$$\frac{2}{1}, \frac{4}{2}, \frac{8}{6}, \frac{16}{24}, \frac{32}{120}, \frac{64}{720}, \frac{128}{5040}, \dots$$

$$r = \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}, \frac{2}{7}, \dots$$

Ratio Test

$\sum a_n$ converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

$\sum a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$

if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$ then the ratio test is **inconclusive**

(note, this test works great for factorials) ex) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(terms are always positive so | | is not necessary)

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}}$$

$$= \lim_{n\to\infty} \left(\frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right)$$

$$= \lim_{n\to\infty} \frac{2}{n+1} = 0$$

Therefore, by the ratio test, the series converges.

You try!

1) $\sum_{n=1}^{\infty} \frac{n^2 \cdot 2^{n+1}}{3^n}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)^2 \cdot 2^{n+2}}{3^{n+1}}}{\frac{n^2 \cdot 2^{n+1}}{3^n}}$$

$$= \lim_{n\to\infty} \frac{(n+1)^2 \cdot 2^{n+2}}{3^{n+1}} \cdot \frac{3^n}{n^2 \cdot 2^{n+1}}$$

$$= \lim_{n\to\infty} \frac{2(n+1)^2}{3n^2}$$

$$= \lim_{n\to\infty} \frac{4(n+1)}{6n} = \frac{2}{3} < 1$$

This series converges based on the ratio test.

2) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}}$$

$$= \lim_{n\to\infty} \left(\frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} \right)$$

$(n+1)\,n!$

$(n+1)(n+1)$

$$\lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1) \cdot n^n}$$

$$= \lim_{n\to\infty} \frac{(n+1)^n}{n^n}$$

$$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n$$

$$= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e > 1$$

This series diverges based on the ratio test.

Brain Break!

It's 'Bad Joke Wednesday'!!

Root Test

(not an AP topic but comes in handy sometimes)

$\sum a_n$ converges absolutely if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$

$\sum a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$

if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$ then the root test is inconclusive

When would this test work well?

ex) $$\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$$

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{e^2}{n}\right)^n}$$

$$= \lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$$

Therefore the series converges based on the root test.

You try!

1) $$\sum_{n=1}^{\infty}\left(\frac{4n+3}{2n-1}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{4n+3}{2n-1}\right)^n}=\lim_{n\to\infty}\frac{4n+3}{2n-1}=2>1$$

Therefore the series diverges based on the root test.

2) $$\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{\ln n}{n}\right)^n}=\lim_{n\to\infty}\frac{\ln n}{n}=\lim_{n\to\infty}\frac{1}{n}=0<1$$

Therefore the series converges based on the root test.

Status check!

Let's see where we're at as far as knowing and understanding all of the convergence tests we've learned so far.

Please log on to goformative.com and take the assessment, "Where are we at?"

This will help drive our focus tomorrow when we begin reviewing for the test next week.

Can we put it all together?

Let's see!

It's time for problems around the room!!

What have we learned?

- Can I apply the ratio test for convergence?
- Can I apply the root test for convergence?

#33 help for Review Quiz #3

How to derive the derivative of an inverse

Suppose $g(x) = f^{-1}(x)$

Then $f(g(x)) = x$

So $f'(g(x)) \cdot g'(x) = 1$

and $g'(x) = \frac{1}{f'(g(x))}$

#33

$f(x) = x^2 - x$ find $g'(10)$

$g'(10) = \frac{1}{f'(g(10))}$

to find $g(10)$, remember that if $g(10) = a$

then $f(a) = 10$

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