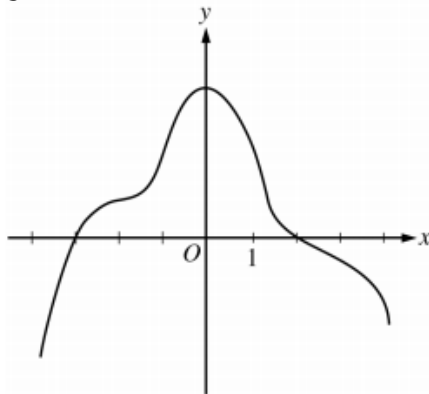


## Let's Get Moving with these AP BC MC Calculator ?s

Graph of  $f'$ 

80. The graph of  $f'$ , the derivative of the function  $f$ , is shown above. Which of the following statements must be true?

- I.  $f$  has a relative minimum at  $x = -3$ .
- II. The graph of  $f$  has a point of inflection at  $x = -2$ .
- III. The graph of  $f$  is concave down for  $0 < x < 4$ .

(A) I only      (B) II only      (C) III only      (D) I and II only      (E) I and III only

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

81. Let  $f$  be a function that is twice differentiable on  $-2 < x < 2$  and satisfies the conditions in the table above. If  $f(x) = f(-x)$ , what are the  $x$ -coordinates of the points of inflection of the graph of  $f$  on  $-2 < x < 2$ ?

- (A)  $x = 0$  only
- (B)  $x = 1$  only
- (C)  $x = 0$  and  $x = 1$
- (D)  $x = -1$  and  $x = 1$
- (E) There are no points of inflection on  $-2 < x < 2$ .

82. What is the average value of  $y = \sqrt{\cos x}$  on the interval  $0 \leq x \leq \frac{\pi}{2}$ ?

- (A) -0.637      (B) 0.500      (C) 0.763      (D) 1.198      (E) 1.882

# Warmup!!

1) Determine convergence or divergence for the following series. If possible, find the sum.

$$\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \quad \checkmark = \frac{1}{1 - -\frac{1}{2}} = \frac{2}{3}$$

*converge, geometric ( $r = -\frac{1}{2}$ )*

2) Approximate the sum of the following series by its first six terms (calculators permitted)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n!}\right)$$

$$\checkmark 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720} = \frac{91}{144} \approx 0.63194$$

$\left(\frac{1}{2}\right)$

540

## 9-5b: Alternating Series Remainder!

### Essential Learning Targets

- I can use the alternating series error bound to estimate how close a partial sum is to the value of an infinite series

## Alternating Series Remainder

If a convergent alternating series has terms where  $a_{n+1} \leq a_n$ , then the absolute value of the remainder,  $R_N$ , of an approximation,  $S_N$ , of a total sum,  $S$ , is  $\leq$  the first neglected term.

$$|S - S_N| = |R_N| \leq a_{n+1}$$

true sum

sum of  
first N  
terms

remainder

first term not  
included in the sum  
(first neglected  
term)

ex) Let's look at the 2nd warmup problem.

We used the first six terms to approximate the sum. By the alternating series remainder theorem, the remainder would be less than or equal to the 7th term.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{1}{n!} \right)$$

$$S_6 = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720} = \frac{91}{144} \approx 0.63194$$

$$a_7 = \frac{1}{7!} = \frac{1}{5040} \approx 0.0002$$

← remainder  $a_7$

↑  
partial sum =  $S_6$

So the true sum,  $S$ , lies between

$0.63194 - 0.0002$  and  $0.63194 + 0.0002$ .

So we get  $0.63174 \leq S \leq 0.63214$

You try! Hmmm, see if you can figure this out.

Determine the number of terms required to approximate the sum of the series with an error less than 0.0001. Then approximate the sum using this many terms. (calculators permitted)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} = \frac{1}{\sqrt{e}}$$

← S

All we really need to find is which term is  $< 0.0001$ . By entering  $a_n$  into y1 and looking at the table, my calculator showed that the  $a_6 \approx 0.00002 < 0.0001$ , so it would take 6 terms ( $a_0 - a_5$ ). The approximate sum would be:

$$\overset{a_0}{1} - \overset{a_1}{\frac{1}{2}} + \overset{a_2}{\frac{1}{8}} - \overset{a_3}{\frac{1}{48}} + \overset{a_4}{\frac{1}{384}} - \overset{a_5}{\frac{1}{3840}} \approx 0.60651$$

Check this with the given actual sum which  $\approx 0.60653$  so we're good!

Brain Break!

It's 'Getting to know you Tuesday'!!

Who will we get to know today??

Evaluate  $\sum_{n=1}^k \frac{(-3)^n (n+2)^4}{2n!}$  such that its value is no more or less than 0.01 from the value of  $\sum_{n=1}^{\infty} \frac{(-3)^n (n+2)^4}{2n!}$

$$\left| \frac{(-3)^n (n+2)^4}{2n!} \right| < 0.01$$

$$\frac{(3)^n (n+2)^4}{2n!} < 0.01$$

Using the calculator, it is the 18th term that falls below 0.01, so  $k = 17$  and we would need to find the sum of the first 17 terms. Using the calculator again, the sum  $\approx -7.531078$ .

On the TI-89, hit F3 and choose the sigma.

Enter  $\Sigma(Y1(x), x, 1, 17)$

On the TI-84, hit math, 0

OR alpha, window, 2 OR 2nd, Stat, math, 5

Sum(seq(Y1, x, 1, 17))

$$\sum_{x=1}^{17} Y1$$



## What have we learned?

- Can I calculate the remainder of an approximation of the sum of an alternating series?