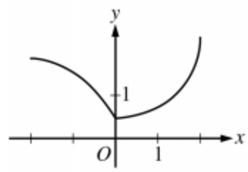
Let's Do This...These are AP BC MC Calculator Questions



Graph of f

76. The function f, whose graph is shown above, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

77. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = x^4$. On what intervals is the rate of change of $f(x)$ greater than the rate of change of $g(x)$?

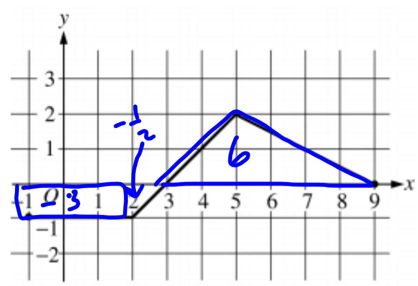
(A) $(0.831, 7.384)$ only

(B) $(-\infty, 0.831)$ and $(7.384, \infty)$

(C) $(-\infty, -0.816)$ and $(1.430, 8.613)$

(D) $(-0.816, 1.430)$ and $(8.613, \infty)$

(E) $(-\infty, \infty)$



Graph of f

78. The graph of the piecewise linear function f is shown above. What is the value of $\int_{-1}^{9} (3f(x) + 2)\, dx$?

(A) 7.5 (B) 9.5 (C) 27.5 (D) 47 (E) 48.5

$$\int_{-1}^{9} 3f(x)\,dx + \int_{-1}^{9} 2\,dx$$

$$3\left(\frac{5}{2}\right) + 2x\Big|_{-1}^{9}$$

$$\frac{15}{2} + 18 - -2 = \frac{55}{2}$$

9-5: Alternating Series!

Essential Learning Targets

- In addition to examining the limit of the sequence of partial sums of a series, I can use a variety of methods for determining convergence or divergence including the nth term test, comparison test, limit comparison test, integral test, ratio test and alternating series test
- I can know that a series may be absolutely convergent, conditionally convergent, or divergent
- I can know that if a series converges absolutely, then it converges

Alternating Series Test

Suppose $a_n > 0$.

If $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$ for all n, then

a_n is nonincreasing

$$\sum_{n=1}^{\infty}(-1)^n a_n \text{ and } \sum_{n=1}^{\infty}(-1)^{n+1} a_n \text{ both converge}$$

In other words, if a_n is positive, nonincreasing and the limit approaches 0, then the alternating series will converge.

ex) $$\sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}}$$

Can you write this as an alternating series?

Think of this as:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{2^{n-1}}$$

So a_n is positive and decreasing for all n

$$\lim_{n\to\infty} \frac{n}{2^{n-1}} = \lim_{n\to\infty} \frac{1}{(2^{n-1})(\ln 2)} = 0$$

Therefore by the alternating series test, the series converges.

You try!

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n}\right)$$

a_n is positive and decreasing for all n

$$\lim_{n\to\infty}\frac{1}{n}=0$$

So the series converges!

You try again!

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$$

$$a_n = \frac{n+1}{n} > 0 \text{ for all } n \quad ✓$$

$$\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} 1 = 1 \neq 0$$

Therefore the alternating series test is inconclusive. 😠

b/c $\lim_{n\to\infty} (-1)^{n+1} \frac{(n+1)}{n} \neq 0$

this diverges by n^{th} term test

Brain Break!

Mathematician Monday!

The first mathematician we will learn about on Mathematician Monday is currently ranked as the #1 mathematician in the world.

Meet Terence Tao, born July 17, 1975.

Terence's father is a pediatrician and his mother is a high-school teacher who teaches math and physics. Both parents studied (and met) at the University of Hong Kong, but moved to Australia after college. Terence has 2 brothers who still live in Australia, one who is working on the GO programming language and the other is an international master in chess. Terence lives with his wife (a NASA engineer) and his 2 kids in Los Angeles.

Terence is one of only 2 people to score a 700 or higher on the math portion of the SAT when he was just 9 years old (he scored a 760). When he was 10 he began competing at the International Math Olympiad (winning a bronze medal) and became the youngest winner ever of the gold medal at this event when he was 13. Terence earned his master's degree in Australia when he was 16 and his PHD from Princeton when he was 20. He began working at UCLA and is still on faculty there.

Terence enjoys collaborating with other mathematicians around the world in order to make new discoveries. It is said that he is only the 2nd person ever (Hilbert being the first) to know and understand 'all' of mathematics. He has published over 300 papers and 17 books. He has a number of original theorems and has won more awards than I can count.

The Green-Tao Theorem (2004): Tao worked with Cambridge mathematician Ben Green to create this theorem which proves that it is always possible to find, somewhere in the infinity of integers, a progression of prime numbers of equal spacing of any length. (3, 7, and 11 are equally spaced by 4)

Absolute vs Conditional Convergence

Absolute Convergence

If $\sum|a_n|$ converges, then $\sum a_n$ converges and is called absolutely convergent.

Conditional Convergence

If $\sum|a_n|$ diverges, but $\sum a_n$ converges, then $\sum a_n$ is called conditionally convergent.

Is the series below absolutely convergent, conditionally convergent, or divergent?

$$ex) \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

$$\text{Check on } \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

Since $\sum|a_n|$ diverges based on the p-test, we need to check the alternating series.

By the alternating series test the original series converges, so it is conditionally convergent.

You try! $\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n(n+1)}{2}}}{3^n}$

Check $\sum_{n=1}^{\infty} \frac{1}{3^n}$

This converges because it is a geometric series with $|r| < 1$. Therefore the original series is absolutely convergent.

You try again!

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$$

$$\text{Check } \sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$$

Let's use the Direct Comparison Test!

Let $b_n = \frac{1}{n}$

$$\frac{1}{n} \leq \frac{1}{\ln(n+1)}$$

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{\ln(n+1)}} = \lim_{n\to\infty} \frac{\ln(n+1)}{n} = 0$$

We know by the p-test that b_n diverges. Therefore the absolute value series diverges as well.

Let's check the alternating series.

Because the series is positive and decreasing, and the limit approaches 0, the original series converges based on the alternating series test. Therefore the series is conditionally convergent.

What have we learned?

- Can I apply the alternating series test to determine if an alternating series converges?
- Can I determine if an alternating series is absolutely convergent, conditionally convergent, or divergent?