Warmup!!

Find the sums of the convergent series below:

1) $\sum_{n=0}^{\infty} 5\left(-\frac{2}{5}\right)^n = \frac{5}{1+\frac{2}{5}} = \frac{25}{7}$

2) $\sum_{n=3}^{\infty} -2\left(\frac{3}{4}\right)^n = \sum_{n=0}^{\infty} -\frac{27}{32}\left(\frac{3}{4}\right)^n = \frac{-\frac{27}{32}}{1-\frac{3}{4}} = -\frac{27}{8}$

3) $\sum_{n=1}^{\infty} \frac{3}{5}\left(-\frac{2}{3}\right)^{n+2}$ $\sum_{n=3}^{\infty} \frac{3}{5}\left(-\frac{2}{3}\right)^n = \sum_{n=0}^{\infty} -\frac{8}{45}\left(-\frac{2}{3}\right)^n = \frac{-\frac{8}{45}}{1+\frac{2}{3}} = -\frac{8}{75}$

4) $\frac{5}{2} - \frac{15}{14} + \frac{45}{98} - \frac{135}{686} + \cdots = \sum_{n=0}^{\infty} \frac{5}{2}\left(-\frac{3}{7}\right)^n = \frac{\frac{5}{2}}{1+\frac{3}{7}} = \frac{7}{4}$

5) $\sum_{n=3}^{\infty} \frac{1}{n^2-n} = \sum_{n=3}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n}\right)$

$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \cdots = \frac{1}{2} - \lim_{n\to\infty} \frac{1}{n} = \frac{1}{2}$

9-3: Integral Test and P-Test for Convergence!

At the end of this lesson you will be able to:

- use the Integral test to determine convergence of series
- Recognize a p-series and determine convergence

The Integral Test!

If f is positive, continuous, and decreasing for $x \geq 1$

and $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \quad and \quad \int_1^{\infty} f(x)dx$$

either BOTH CONVERGE or BOTH DIVERGE

ex) Determine the convergence/divergence of

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

f(x) is positive ✓

f(x) is continuous ✓

f(x) is decreasing ✓

(if this is not obvious, use f ' to check)

$$\int_1^{\infty} \frac{1}{x^2+1}dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{x^2+1}dx$$

$$= \lim_{b\to\infty} (\arctan b - \arctan 1)$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

You try! Use the integral test to determine the convergence or divergence of each series

1) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3n+5}$

f is positive
f is continuous
f is decreasing

$$\lim_{b\to\infty}\int_1^b \frac{2}{3x+5}\,dx = \lim_{b\to\infty}\frac{2}{3}\left(\ln|3b+5| - \ln|8|\right) = \infty$$

so the series diverges

2) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

f is positive, continuous and decreasing

$$\lim_{b\to\infty}\int_1^b \frac{\ln x}{x^3}\,dx = \lim_{b\to\infty}\left(-\frac{1}{4x^2} - \frac{\ln x}{2x^2}\Big|_1^b\right)$$

$$\lim_{b\to\infty}\left(-\frac{1}{4b^2} - \frac{\ln b}{2b^2} + \frac{1}{4}\right) = 0 - 0 + \frac{1}{4} = \frac{1}{4}$$

so the series converges

3) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{e^n}$

f is positive, co... hmmm, is f positive? hmmm, nope. Integral test does not apply.

P-Series Convergence Test!

The p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$$

converges if $p > 1$

diverges if $0 < p \leq 1$

Determine if the following series converge or diverge.

1) $\sum_{n=1}^{\infty} \frac{3}{n^{\frac{5}{3}}}$ p = 5/3 > 1 so the series converges

2) $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \cdots$

p = 2/3 < 1 so the series diverges

Can you put it all together?

Determine convergence or divergence of the following series.

1) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}$	This one can't be determined yet with the tests that we know (integral test does not apply because f(x) is discontinuous at x = 1)
2) $\sum_{n=1}^{\infty} \frac{3}{n^{0.95}}$	diverges -> p-test, $p \leq 1$
3) $\sum_{n=0}^{\infty} (1.075)^n$	diverges -> geometric sequence with $r > 1$
4) $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3}\right)$	converges -> p-test (converge ± converge = converge)
5) $\sum_{n=2}^{\infty} \ln n$	$\lim_{n\to\infty} \ln n = \infty$ diverges based on n^{th} term test

Quiz 9.2

1) (NC) Express the repeating decimal, $.38313\overline{31}$ as a geometric series and then as a fraction of 2 integers

2) (NC) Determine the convergence or divergence of each of the following series. Justify your answers. If the series converges, find the sum.

a) $\sum_{n=0}^{\infty}\left(\frac{5}{2^n}-\frac{1}{3^n}\right)$

b) $\sum_{n=0}^{\infty}\frac{(-1)^n 5}{4^n}$

c) $\sum_{n=0}^{\infty}\frac{2^n-1}{3^n}$

d) $\sum_{n=1}^{\infty}\frac{1}{(4n-3)(4n+1)}$

3) (C) Upon graduation from college you receive two job offers. One, with a starting salary of $28,000 on which you will receive a 5.2% raise every year. The second, with a starting salary of $44,000 on which you will receive a 3.7% raise every year. After 45 years, by how much more will the total earned salary of one differ from the other? (Use geometric series to solve.)

What have we learned?

- Can I apply the integral test to determine convergence?
- Can I apply the p-test to determine convergence?
- Can I apply all of the other tests I've learned (nth term, geometric series) and know which test to apply to determine convergence?

March 02, 2016