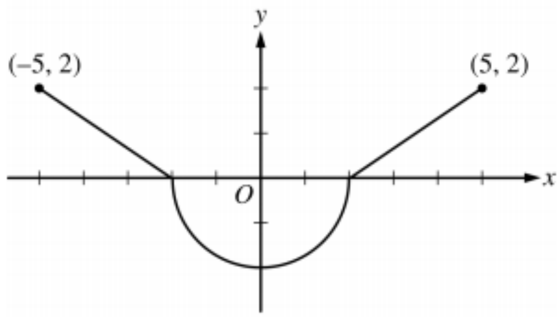
Let's Warm Up Those Brain Cells with these 2012 AP BC No Calculator Questions



Graph of f'

18. The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2}, \dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \dfrac{2}{3}\right)$

(E) There are no such points.

20. $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

(A) $\ln(8)$ (B) $\ln\left(\dfrac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

Warmup!!

A pendulum is released to swing freely. On the first swing, the pendulum travels a distance of 18 inches. On each successive swing, the pendulum travels 90% the distance of the previous swing. What is the total distance the pendulum swings?

$$\sum_{n=0}^{\infty} \underline{18}\underbrace{(0.9)}_{r}^{n} = \frac{18}{1-0.9} = 180\ inches$$

9-2b: More Series and Convergence!

Essential Learning Targets

- know that the nth partial sum is the sum of the first n terms of a sequence
- know that an infinite series converges to a real number, S, if and only if the limit of its sequence of partial sums exists and equals S
- recognize common series including geometric series

2nd Warmup!

See if you can figure out how to write each repeating decimal as the sum of an infinite series and convert the decimal to a fraction. (An example is shown below to help you.)

ex) 0.777777... = .7 + .07 + .007 + .0007 + ...

$= 0.7 + 0.7(1/10) + 0.7(1/10)^2 + 0.7(1/10)^3 + \dots$

$$\sum_{n=0}^{\infty} \frac{7}{10}\left(\frac{1}{10}\right)^n = \frac{7}{9}$$

You try) 0.818181818181818181...

$0.81818181\ldots = 0.81 + 0.0081 + 0.000081 + \cdots$

$= .81\cancel{(1/100)} + .81(1/100)^{1} + .81(1/100)^{2} + \dots$

$$\sum_{n=0}^{\infty} \frac{81}{100}\left(\frac{1}{100}\right)^n = \frac{81}{99} = \frac{9}{11}$$

Determine the convergence or divergence of the series. Find the sum, if possible.

$$\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

$$\sum_{n=2}^{\infty} \frac{1}{100}\left(\frac{1}{5}\right)^{n}$$

Work in your groups on these.

Very common MC problem: Find all values of x for which the series converges. Then write the sum of the series in terms of x.

For a series of the form x^n to converge, $|x| < 1$.

1) $\sum_{n=1}^{\infty} (3x)^n$

r

$|3x| < 1$

$-1 < 3x < 1$

$-\frac{1}{3} < x < \frac{1}{3}$

We know that $|3x| < 1$, so $|x| < 1/3$

So $-1/3 < x < 1/3$

$$\sum_{n=1}^{\infty} (3x)^n = \sum_{n=0}^{\infty} (3x)(3x)^n = \frac{3x}{1-3x}$$

2) $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

$$(-1)^n x^{2n} = (-1)^n (x^2)^n = (-x^2)^n$$

so $|-x^2| < 1$ which is the same as $x^2 < 1$

solving the inequality (use a sign line) we get $-1 < x < 1$

$$\sum_{n=0}^{\infty} (-1)^n x^{2n} = \sum_{n=0}^{\infty} (-x^2)^n = \frac{1}{1+x^2}$$

3) $\sum_{n=1}^{\infty} \left(\frac{x^2}{x^2+4}\right)^n$

$\left|\frac{x^2}{x^2+4}\right| < 1$ *which is true for all x*

$(-\infty, \infty)$

$$\sum_{n=0}^{\infty} \left(\frac{x^2}{x^2+4}\right)\left(\frac{x^2}{x^2+4}\right)^n$$

$$= \frac{\frac{x^2}{x^2+4}}{1 - \frac{x^2}{x^2+4}} = \frac{x^2}{x^2+4-x^2} = \frac{x^2}{4}$$

1993 #27

The interval of convergence for $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is:

a) $-3 < x \leq 3$

b) $-3 \leq x \leq 3$

c) $-2 < x < 4$ ✓

d) $-2 \leq x < 4$

e) $0 \leq x \leq 2$

Answer is C!!

$$\sum_{n=0}^{\infty} \left(\frac{x-1}{3}\right)^n$$

$$\left|\frac{x-1}{3}\right| < 1$$

$$-1 < \frac{x-1}{3} < 1$$

$$-3 < x-1 < 3$$

$$-2 < x < 4$$

Brain Break!

It's Bad Joke Wednesday!!

6 HELPFUL LIMITS

In your groups, see how many of these you can figure out.

1) $\lim\limits_{n\to\infty} \dfrac{\ln n}{n} = 0$

2) $\lim\limits_{n\to\infty} x^{\frac{1}{n}} = 1$ for $x > 0$

3) $\lim\limits_{n\to\infty} \left(1+\dfrac{x}{n}\right)^n = e^x$

4) $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$

5) $\lim\limits_{n\to\infty} x^n = 0$ for $|x| < 1$, ∞ for $x > 1$

6) $\lim\limits_{n\to\infty} \dfrac{x^n}{n!} = 0$

3) $\ln y = \lim\limits_{n\to\infty} n \ln\left(1+\frac{x}{n}\right)$

$$= \lim_{n\to\infty} \frac{\ln\left(1+\frac{x}{n}\right)}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{\dfrac{\frac{-x}{n^2}}{1+\frac{x}{n}}}{-\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{\frac{-x}{n^2}}{\frac{n+x}{n}} \cdot \frac{-n^2}{1}$$

$$= \lim_{n\to\infty} \frac{\cancel{-}xn}{\cancel{n}(n+x)} \cdot \cancel{-}n^{\cancel{2}} = \lim_{n\to\infty} \frac{xn}{x+n} = \lim_{n\to\infty} \frac{x}{1}$$

$$= \frac{x}{1}$$

$\ln y = x$ $\boxed{y = e^x}$

Determine if the following sequences converge or diverge.

1) $a_n = \dfrac{\sin^2 n}{2^n}$ converges to 0

2) $a_n = \dfrac{\ln n}{\sqrt{n}}$ converges to 0

3) $a_n = (-1)^n \left(1 - \dfrac{1}{n}\right)$ diverges

4) $a_n = \dfrac{n + (-1)^n}{n}$ converges to 1

5) $a_n = \dfrac{\ln n}{\ln(2n)}$ converges to 1

6) $a_n = \dfrac{(n+1)!}{n!}$ diverges

7) $a_n = \dfrac{n^2 - n}{2n^2 + n}$ converges to 1/2

8) $a_n = \dfrac{2n + \sin n}{n + \cos(5n)}$ converges to 2

The annual spending by tourists in a resort city is \$100 million. Approximately 75% of that revenue is again spent in the resort city, and of that amount, approximately 75% is again spent in the same city, and so on. Write the geometric series that gives the total amount of spending generated by the \$100 million and find the sum of the series after 10 years.

(in millions)

$$\sum_{n=0}^{9} 100(.75)^n = \left(\frac{100}{1-.75}\right)\left(1-(.75)^{10}\right)$$

$$\approx \$377.474 \text{ mil}$$

$$\sum_{n=0}^{K} a(r)^n = \left(\frac{a}{1-r}\right)\left(1-r^{K+1}\right)$$

A deposit of \$1000 is made at the end of each year for 20 years into an account that pays 4.25% interest, compounded yearly. Determine the balance of the account at the end of 20 years.

$$\sum_{n=0}^{19} 1000(1.0425)^n = \left(\frac{1000}{1-1.0425}\right)\left(1-(1.0425)^{20}\right)$$

$$\approx \$30562.50$$

What have we learned?

- What is the condition for a series in the form x^n to converge?
- Can I write a repeating decimal as a series and find an equivalent fraction?