Let's Warm Up with these 2012 BC MC No Calc Questions...

14. Let k be a positive constant. Which of the following is a logistic differential equation?

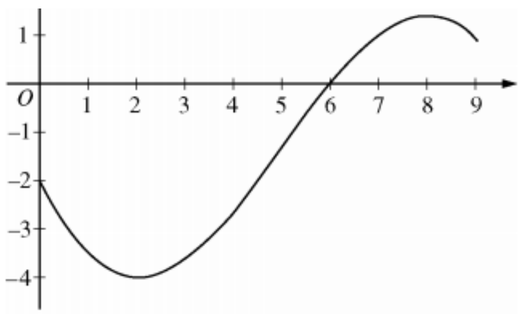
(A) $\dfrac{dy}{dt} = kt$

(B) $\dfrac{dy}{dt} = ky$

(C) $\dfrac{dy}{dt} = kt(1-t)$

(D) $\dfrac{dy}{dt} = ky(1-t)$

(E) $\dfrac{dy}{dt} = ky(1-y)$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

16. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\dfrac{5}{4}$ (B) 1 (C) $\dfrac{7}{4}$ (D) 2 (E) $\dfrac{21}{4}$

x	y
1	3
$\frac{3}{2}$	
2	

Warmup!

Note: a <u>partial sum</u>, S_n, is equal to the sum of the first n terms of a series.

Find the first 5 partial sums of each series below. (calculators permitted)

$S_1 = 1,\ S_2 = 4,\ S_3 = 9, \ldots$

1) $1 + 3 + 5 + 7 + 9 + \ldots$ ✓ 1, 4, 9, 16, 25

2) $$\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} + \cdots$$

✓ 1/6, 1/4, 3/10, 1/3, 5/14

3) $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$$

$a_1 = 1,\ a_2 = \frac{-1}{2},\ a_3 = \frac{1}{6},\ a_4 = \frac{-1}{24}$

$a_5 = \frac{1}{120}$

✓ 1, 1/2, 2/3, 5/8, 19/30

9-2a: Series and Convergence!

Essential Learning Targets

- know that the nth partial sum is the sum of the first n terms of a sequence
- know that an infinite series converges to a real number, S, if and only if the limit of its sequence of partial sums exists and equals S
- recognize common series including geometric series

Definition: if the sequence of partial sums of a series converges, then the infinite sum of the series converges. The convergence value is equal to the sum of the series.

nth Term Test for Divergence:

$$\text{If } \lim_{n\to\infty} a_n \neq 0, \text{then } \sum_{n=1}^{\infty} a_n \text{ diverges}$$

Note that the converse of this is not necessarily true. Just because this limit equals 0 does not imply that the series will converge.

Determine if the following series diverge.

1) $\sum_{n=1}^{\infty} 1$

$$\lim_{n\to\infty} 1 = 1$$

diverges based on nth-term test

2) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

$$\lim_{n\to\infty} \left(\frac{3}{2}\right)^n = \infty$$

diverges based on nth-term test

3) $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

hmmm, can't be determined based on the test - we'll have to find another way (we'll save this for section 9-3)

Geometric Series Test

If the series is <u>geometric</u> (of the form $\sum_{n=0}^{\infty} a \cdot r^{n}$)

then the series will converge to $\dfrac{a}{1-r}$ as long as

$|r| < 1$

"common ratio"

$|r| < 1 \Rightarrow$ converges

$|r| \geq 1 \Rightarrow$ diverges

Note: this is for the sum from n = 0 to infinity, so if you have a sum starting with any other n, you will need to convert it to a sum starting with n = 0 (or subtract the zeroith term).

Ex) Rewrite as a sum from n = 0. Then find the sum.

$$\sum_{n=4}^{\infty} 45\left(-\frac{2}{3}\right)^{n}$$

$r = -\frac{2}{3}$
so converges
by geometric

a) Find the 'first term' (at n = 4)

$$45\left(-\frac{2}{3}\right)^{4} = 45\left(\frac{16}{81}\right) = \frac{80}{9}$$

b) Rewrite the sum

$$\sum_{n=0}^{\infty} \frac{80}{9}\left(-\frac{2}{3}\right)^{n}$$

c) Find the sum

$$\frac{\frac{80}{9}}{1 - -\frac{2}{3}} = \frac{80}{9} \cdot \frac{3}{5} = \frac{16}{3}$$

Show that the following series converge and find the value of convergence for each

1) $\sum_{n=0}^{\infty} \frac{1}{2^n}$

geometric, $r = \frac{1}{2}$

✓converges to 2

2) $\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} -1\left(-\frac{1}{2}\right)^n$

geometric, $r = -\frac{1}{2}$

✓ converges to -2/3

$\text{Sum} = \frac{-1}{1 - -\frac{1}{2}}$

3) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$

$a = 1 \quad r = \frac{2}{3}$

$\sum_{n=0}^{\infty} 1\left(\frac{2}{3}\right)^n$

geometric, $r = \frac{2}{3}$

✓converges to 3

Shall we take it up a notch? Determine if the following series converge or diverge. If they converge, find the sum.

$$\sum_{n=2}^{\infty} -\frac{2}{5}\left(\frac{1}{3}\right)^n = \sum_{n=0}^{\infty} -\frac{2}{45}\left(\frac{1}{3}\right)^n = \frac{-\frac{2}{45}}{1-\frac{1}{3}} = -\frac{1}{15}$$

converges to -1/15 (geometric, $r=\frac{1}{3}$)

$$\sum_{n=1}^{\infty} \frac{7(4^n)}{(2^n)(5^n)} = \sum_{n=0}^{\infty} \frac{14}{5}\left(\frac{2}{5}\right)^n = \frac{\frac{14}{5}}{1-\frac{2}{5}} = \frac{14}{3}$$

converges to 14/3

$$\frac{4^n}{2^n \cdot 5^n} = \frac{4^n}{10^n} = \left(\frac{4}{10}\right)^n$$

Brain Break!

Today is Tuesday, you know what that means, we're going to have a special guest today! And our guest is...

Telescoping Form

Consider the series: $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

If we list out the first several terms of the sequence we get:

$$1 - \frac{1}{2}, \quad \frac{1}{2} - \frac{1}{3}, \quad \frac{1}{3} - \frac{1}{4}, \quad \frac{1}{4} - \frac{1}{5}, \ldots$$

So our partial sums would be as follows:

$$\underline{1} \cancel{- \frac{1}{2} + \frac{1}{2}} \cancel{- \frac{1}{3} + \frac{1}{3}} \cancel{- \frac{1}{4} + \frac{1}{4}} \cancel{- \frac{1}{5}} + \ldots \cancel{+ \frac{1}{\infty}} \underline{- \frac{1}{\infty - 1}}$$

This is called a <u>telescoping series</u> because all of the middle terms "collapse" so that each sum is just a combination of the first part of the first term and the last part of the last term

If the series converges, the sum will often be as follows: $\sum_{n=1}^{\infty} a_n - b_n = a_1 - \lim_{n\to\infty} b_n$

Always write out the first several terms to make sure.

So for our example, the final sum is:

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1} = 1 - \lim_{n\to\infty} \frac{1}{n+1} = 1 - 0 = 1$$

$$ex)\ \sum_{n=1}^{\infty} \frac{2}{4n^2-1}$$

Use partial fractions to break down a_n

$$\frac{2}{4n^2-1} = \frac{2}{(2n+1)(2n-1)} = \frac{A}{2n+1} + \frac{B}{2n-1}$$

$$= -\frac{1}{2n+1} + \frac{1}{2n-1} = \frac{1}{2n-1} - \frac{1}{2n+1}$$

Check to make sure this is a telescoping series by listing out the first several terms of the nth partial sum:

$$1 - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{9}} + \cdots \cancel{\frac{1}{\infty-1}} - \frac{1}{\infty+1}$$

Since this is telescoping, the sum will equal:

$$1 - \lim_{n\to\infty} \frac{1}{2n+1} = 1 - 0 = 1$$

So this series converges to 1

You try! Determine convergence and find the sum of the series below:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$$

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2n} - \frac{1}{2(n+2)}\right)$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{1}{4} - \frac{1}{8} + \frac{1}{6} - \frac{1}{10} + \frac{1}{8} - \frac{1}{12} + \cdots$$

$$\frac{1}{2} + \frac{1}{4} - \lim_{n \to \infty} \frac{1}{2(n+2)} = \frac{3}{4}$$

What have we learned?

- What are partial sums?
- What does it mean for a series to converge?
- What is the nth-term test for divergence?
- Is a telescoping series one that requires a telescope to see?
- How do we rewrite a series in telescoping form?