Warmup!

Simplify each of the following:

1) $\frac{25!}{23!} = \frac{25 \cdot 24 \cdot 23!}{23!} = 25 \cdot 24 = 600$

2) $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)(n!)}{n!} = (n+2)(n+1)$

3) $\frac{(2n+2)!}{(2n)!} = \frac{(2n+2)(2n+1)(2n)!}{(2n)!} = (2n+2)(2n+1)$

Remember 5! = 5 · 4 · 3 · 2 · 1 = 120

5! is called '5 factorial'

Chapter 9

Infinite Series!

9-1a: Sequences

At the end of this lesson you will be able to:

- Write the terms of a sequence
- Find the limit of a sequence
- Determine convergence of a sequence

What is a sequence?

A sequence is a function whose domain is the set of all positive integers. We usually write sequences with the notation a_n denoting the n^{th} term of the sequence.

So for the sequence: 2, 4, 6, 8, 10, ... we would write $a_1 = 2$, $a_2 = 4$, $a_3 = 6$, $a_4 = 8$, $a_5 = 10$, ...

↳ the 'first term'

In general, $a_n = 2n$.

A sequence is said to be explicitly defined if the input is an n-value.

A sequence is said to be recursively defined if the input is a previous term of the sequence.

Give these a try!

Find the first 5 terms of each sequence.
Then find the limit of an as n approaches ∞.

1) $a_n = \dfrac{2n}{n+3}$ ✓ $\dfrac{1}{2}, \dfrac{4}{5}, 1, \dfrac{8}{7}, \dfrac{5}{4}$

$$\lim_{n\to\infty} \frac{2n}{n+3} = \lim_{n\to\infty} \frac{2}{1} = 2$$

2) $a_1 = 4, a_{k+1} = \left(\dfrac{k+1}{2}\right) a_k$ ✓ $4, 4, 6, 12, 30$

$$\lim_{n\to\infty} a_n = \infty$$

$a_2 \to K=1$

$a_2 = \left(\frac{1+1}{2}\right) \cdot a_1 = 1(4) = 4$

Finding limits with sequences is the same as finding limits with regular functions. You may still use L'Hopital if the limit approaches 0/0 or $\pm\infty/\pm\infty$.

A sequence is convergent if the limit exists.

A sequence is divergent is the limit does not exist.

Note: when referring to the limit of a sequence, we are referring to the limit as n approaches infinity.

Find the limit of each sequence below. Then state if each sequence converges or diverges.

1) $a_n = \dfrac{n}{1-2n}$

✓ $\lim_{n\to\infty} \dfrac{n}{1-2n} = \lim_{n\to\infty} \dfrac{1}{-2} = -\dfrac{1}{2} \Rightarrow converges$

2) $a_n = \dfrac{\ln n}{n}$

✓ $\lim_{n\to\infty} \dfrac{\ln n}{n} = \lim_{n\to\infty} \dfrac{\frac{1}{n}}{1} = 0 \Rightarrow converges$

3) $a_n = \dfrac{5n}{\sqrt{n^2 + 4}}$

✓ $\lim_{n\to\infty} \dfrac{5n}{\sqrt{n^2+4}} = \lim_{n\to\infty} \dfrac{5n}{n} = 5 \Rightarrow converges$

4) $a_n = \cos\dfrac{2}{n}$

✓ $\lim_{n\to\infty} \cos\dfrac{2}{n} = \cos 0 = 1 \Rightarrow converges$

For each of the following,

a) Write the explicit rule

b) Determine convergence or divergence

1) $-\frac{1}{3}, \frac{1}{4}, \frac{3}{5}, \frac{5}{6}, 1, \ldots$

$a_n = (2n-3)/(n+2)$

converges to 2

2) $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, -\frac{1}{243}, \ldots$

$a_n = (-1)^n/3^n$ or $(-1/3)^n$

converges to 0

3) 1, -1, 1, -1, 1, -1, ...

$a_n = (-1)^{(n-1)}$

diverges

4) $-\frac{2}{3}, \frac{1}{6}, \frac{2}{3}, \frac{13}{12}, \frac{22}{15}, \ldots$

$a_n = (n^2 - 3)/3n$

diverges

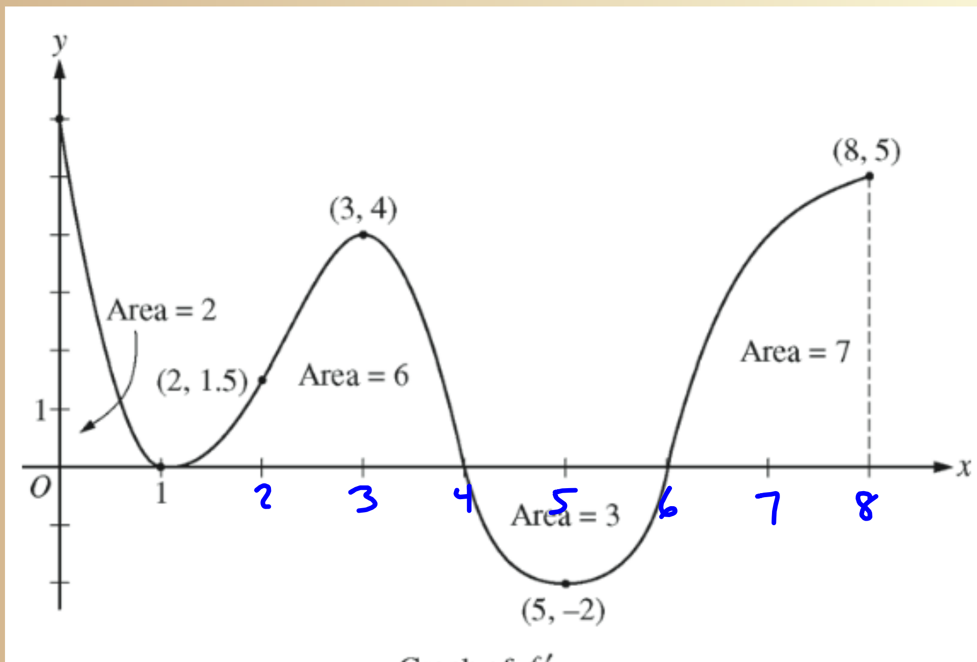
5) $\frac{4}{9}, \frac{2}{3}, 1, \frac{3}{2}, \frac{9}{4}, \ldots$

$a_n = (4/9)(3/2)^{(n-1)}$

diverges

Do we have time for some review? I think so!

BC 2013 #4



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum.

b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing?

d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -5/2$, find the slope of the line tangent to the graph of g at $x = 3$.

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What have we learned?

- What is a sequence?
- What is meant by the limit of a sequence?
- How do we know if a sequence converges or diverges?