2013 BC free response #3 (no calc)

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of C(t), measured in ounces, are given in the table above.

a) Use the data in the table to approximate $\boxed{C'}(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$\frac{12.8-11.2}{4-3} \ \frac{oz}{min}$$

b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer. C is cont/diff $\quad$ sos $= \dfrac{C(4)-C(2)}{4-2} = 2$

c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $1/6 \int_0^6 \boxed{C(t)}dt$. Using correct units, explain the meaning of $1/6 \int_0^6 C(t)dt$ in the context of the problem. $\frac{1}{6} \cdot 2(5.3+11.2+13.8)$

d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when t = 5.

$$B'(t) = -16(-.4)e^{-.4t}$$

$$= -16(-.4)e^{[-.4(5)]}$$

8.6 Integration by Tables

At the end of this lesson you will be able to:

- Evaluate integrals using a table of integrals

The biggest challenge with integrals is recognizing how to approach/attack each one.

Give each of the following a try:

$$\int x \ln x \, dx \qquad = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$u = \ln x \quad du = \frac{1}{x}dx$ this one requires integration by parts

$v = \frac{1}{2}x^2 \quad dv = x\,dx \quad \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}dx$

$$\int \frac{\ln x}{x} dx = \frac{1}{2}(\ln x)^2 + C$$

this one requires u-sub and power rule

$$\int \frac{1}{x \ln x} dx = \ln|\ln x| + C$$

this one requires u-sub and natural log rule

Integration tables are kind of 'cheat sheets' for integrals. The tables can be found in appendix B in your textbook. I've also linked them to my website in case you need to refer to them and don't have your book handy.

Note that you will never be allowed to use these on a test or the ap exam, so this is kind of a fun 'free' day just so that you know they exist.

Integrate the following using tables. Be careful, some still require some manipulating or u-substitution before you can apply the formulas. Some might even require more than one formula!

$$1)\ \int x^2\sqrt{9x^2+2}\,dx = 3\int x^2\sqrt{x^2+\frac{2}{9}}\,dx$$

$$\frac{3}{8}\left[x\left(2x^2+\frac{2}{9}\right)\sqrt{x^2+\frac{2}{9}} - \frac{4}{81}\ln\left|x+\sqrt{x^2+\frac{2}{9}}\right|\right]+C$$

ula 27

$$\Big] + C \qquad \frac{e^x}{1-\tan e^x}\,dx$$

$u = e^x$

$du = e^x dx$

$$= \int \frac{1}{1-\tan u}\,du$$

let u = e^x and apply formula 71

$$= \frac{1}{2}\left(e^x - \ln|\cos e^x - \sin e^x|\right) + C$$

$$3)\ \int x^3 \sin x\,dx$$

$$= -x^3\cos x + 3\int x^2\cos x\,dx$$ formula 54

$$= -x^3\cos x + 3\left(x^2\sin x - 2\int x\sin x\,dx\right)$$ formula 55

$$= -x^3\cos x + 3x^2\sin x - 6(\sin x - x\cos x) + C$$ formula 52

$$\int x^3 \sin x \, dx$$

tab method: polynomial × function that can be integrated repeatedly

	poly. u	other dv
$+$	x^3	$\sin x$
$-$	$3x^2$	$-\cos x$
$+$	$6x$	$-\sin x$
$-$	6	$\cos x$
$+$	0	$\sin x$

$$-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$$

What have we learned?

- Can I look at integral and choose the correct integration formula to use?
- Can I simplify the integral first so that the formula applies correctly?