It's Warmup time!

Evaluate the following integral and <u>simplify completely</u>

$$\int_1^5 \frac{x-1}{x^2(x+1)}\,dx$$

$$\frac{x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

OR

$$= \frac{Ax+B}{x^2} + \frac{C}{x+1}$$

✓

$$\frac{x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$x - 1 = Ax(x+1) + B(x+1) + Cx^2$$

$$let\ x = -1\ to\ get\ C = -2, let\ x = 0\ to\ get\ B = -1$$

$$let\ x = 1\ to\ get\ A = 2$$

$$\int_1^5 \frac{x-1}{x^2(x+1)}\,dx = \int_1^5 \left(\frac{2}{x} - \frac{1}{x^2} - \frac{2}{x+1}\right) dx$$

$$= 2\ln|x| + \frac{1}{x} - 2\ln|x+1| \Big|_1^5$$

$$= 2\ln 5 + \frac{1}{5} - 2\ln 6 - 1 + 2\ln 2$$

$$= (\ln 25 - \ln 36 + \ln 4) - \frac{4}{5} = \ln\frac{100}{36} - \frac{4}{5}$$

$$= \ln\frac{25}{9} - \frac{4}{5}\ or\ 2\ln\frac{5}{3} - \frac{4}{5}$$

HW 8-5a

(25) $\int \frac{x}{16x^4-1}\,dx$

$= \int \frac{x}{(4x^2+1)(4x^2-1)}\,dx$

$= \int \frac{x}{(4x^2+1)(2x+1)(2x-1)}\,dx$

$\frac{x}{(4x^2+1)(2x+1)(2x-1)} = \frac{Ax+B}{4x^2+1} + \frac{C}{2x+1} + \frac{D}{2x-1}$

$x = (Ax+B)(2x+1)(2x-1) + C(4x^2+1)(2x-1) + D(4x^2+1)(2x+1)$

let $x=\frac{1}{2}$, $\frac{1}{2} = D(2)(2)$ so $D = \frac{1}{8}$

let $x=-\frac{1}{2}$, $-\frac{1}{2} = C(2)(-2)$ so $C = \frac{1}{8}$

let $x=0$, $0 = B(1)(-1) + \frac{1}{8}(1)(-1) + \frac{1}{8}(1)(1)$

so $B=0$

let $x=1$, $1 = A(3)(1) + \frac{1}{8}(5)(1) + \frac{1}{8}(5)(3)$

$1 = 3A + \frac{5}{8} + \frac{15}{8}$ so $8 = 24A + 20$

$A = -\frac{1}{2}$

$\int \left[\frac{-x}{2(4x^2+1)} + \frac{1}{8(2x+1)} + \frac{1}{8(2x-1)}\right] dx$

$-\frac{1}{16}\ln|4x^2+1| + \frac{1}{16}\ln|2x+1| + \frac{1}{16}\ln|2x-1| + C$

$= \frac{1}{16}\ln\left|\frac{(2x+1)(2x-1)}{4x^2+1}\right| + C$

8.5b Putting Trig, U-Substitution and Partial Fractions together!

At the end of this lesson you will be able to:

- solve problems involving partial fractions and trig substitution
- review a large variety of integrals just to see if you're still keeping everything straight

Jumping right in!

Suppose we need to evaluate $\int \frac{\sin x}{\cos x + \cos^2 x} dx$

Try letting u = cosx and completing the u-sub. Then use partial fractions to rewrite and integrate.

$let\ u = cosx, so\ du = \ -sinxdx$

$$so \int \frac{-1}{u + u^2} du = \int \frac{-1}{u(1+u)} du$$

✓

$$Using\ partial\ fractions\ we\ get: \int \left(\frac{-1}{u} + \frac{1}{1+u}\right) du$$

$$= \ln|1 + u| - \ln|u| + C$$

$$= \ln|1 + \cos x| - \ln|\cos x| + C$$

Try one more of this type!

$$\int \frac{e^x}{(e^x+1)(e^x-9)}dx$$

$$let\ u = e^x, so\ du = e^x dx$$

$$\int \frac{1}{(u+1)(u-9)}du$$

$$\frac{1}{(u+1)(u-9)} = \frac{A}{u+1} + \frac{B}{u-9}$$

✓

$$so\ 1 = (A)(u-9) + B(u+1)$$

$$let\ u = 9\ to\ get\ B = \frac{1}{10}, let\ u = -1\ to\ get\ A = -\frac{1}{10}$$

$$so\ our\ integral\ becomes:$$

$$\int \left(\frac{-\frac{1}{10}}{u+1} + \frac{\frac{1}{10}}{u-9} \right) du = -\frac{1}{10} \int \left(\frac{1}{u+1} - \frac{1}{u-9} \right) du$$

$$= -\frac{1}{10} (\ln|u+1| - \ln|u-9|) + C$$

$$= -\frac{1}{10} \ln\frac{|e^x+1|}{|e^x-9|} + C = \frac{1}{10} \ln\frac{|e^x-9|}{|e^x+1|} + C$$

AP PRACTICE!

BC 2008 Multiple Choice #8

$$\int \frac{1}{x^2 - 6x + 8}\,dx =$$

a) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$ b) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$ c) $\frac{1}{2}\ln|(x-2)(x-4)| + C$

d) $\frac{1}{2}\ln|(x+2)(x-4)| + C$ e) $\ln|(x-2)(x-4)| + C$

BC 2003 Multiple Choice #11

$$\int \frac{2x}{(x+2)(x+1)}\,dx =$$

a) $\ln|x+2| + \ln|x+1| + C$ b) $\ln|x+2| + \ln|x+1| - 3x + C$

c) $-4\ln|x+2| + 2\ln|x+1| + C$ d) $4\ln|x+2| - 2\ln|x+1| + C$

e) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

What have we learned?

Can I:

- use u-substitution in combination with trig and partial fractions to evaluate an integral?