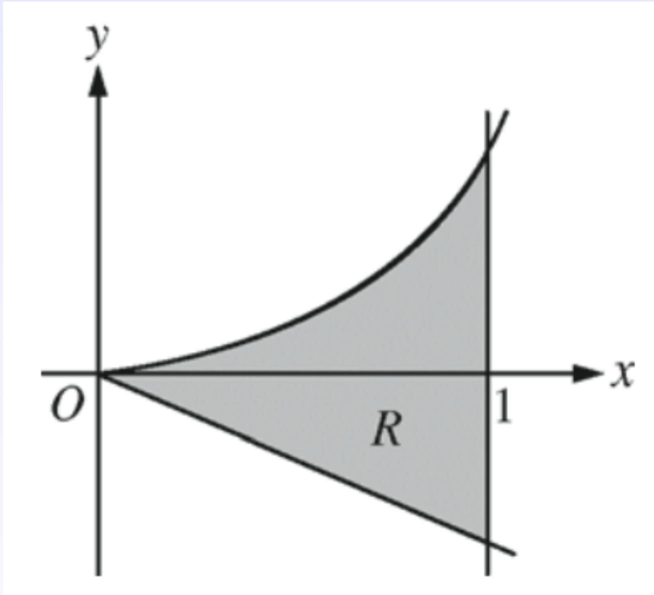
2014 BC #5

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure to the right.



a) find the area of R

b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$

c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R

15 $\int \sin^2\alpha \cos^2\alpha \, d\alpha$

$$= \int \frac{1}{2}(1-\cos 2\alpha)\left(\frac{1}{2}\right)(1+\cos 2\alpha)\, d\alpha$$

$$= \frac{1}{4}\int (1-\cos^2 2\alpha)\, d\alpha$$

$$= \frac{1}{4}\int \left[1-\frac{1}{2}(1+\cos 4\alpha)\right] d\alpha$$

$$= \frac{1}{4}\left[\alpha - \frac{1}{2}\alpha - \frac{1}{8}\sin 4\alpha\right] + C$$

$$= \frac{1}{8}\alpha - \frac{1}{32}\sin 4\alpha + C$$

(17) $\int x \sin^2 x \, dx$

$$= \int x\left(\tfrac{1}{2}\right)(1-\cos 2x)\,dx$$

$$= \frac{1}{2}\int (x - x\cos 2x)\,dx$$

$$= \frac{1}{4}x^2 - \frac{1}{2}\int x\cos 2x\,dx$$

	u	dv
+	x	$\cos 2x$
−	1	$\frac{1}{2}\sin 2x$
+	0	$-\frac{1}{4}\cos 2x$

$$= \frac{1}{4}x^2 - \frac{1}{2}\left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right] + C$$

27 $\int \sec^4(5x)\,dx$

$= \int \sec^2(5x)\left[\tan^2(5x)+1\right]dx$

$= \int \sec^2(5x)\tan^2(5x)\,dx + \int \sec^2(5x)\,dx$

$u = \tan(5x)$

$du = 5\sec^2(5x)\,dx$

$= \int \frac{1}{5}u^2\,du + \int \sec^2(5x)\,dx$

$= \frac{1}{15}u^3 + \frac{1}{5}\tan(5x) + C$

$= \frac{1}{15}\tan^3(5x) + \frac{1}{5}\tan(5x) + C$

29 $\int \sec^3 \pi x \, dx$

$u = \sec \pi x \qquad du = \pi \sec \pi x \tan \pi x \, dx$

$v = \frac{1}{\pi} \tan \pi x \qquad dv = \sec^2 \pi x \, dx$

$$\int \sec^3 \pi x \, dx = \frac{1}{\pi} \sec \pi x \tan \pi x - \int \sec \pi x \tan^2 \pi x \, dx$$

$$= \frac{1}{\pi} \sec \pi x \tan \pi x - \int \sec \pi x (\sec^2 \pi x - 1) \, dx$$

$$= \frac{1}{\pi} \sec \pi x \tan \pi x - \int \sec^3 \pi x \, dx + \int \sec \pi x \, dx$$

$$2 \int \sec^3 \pi x \, dx = \frac{1}{\pi} \sec \pi x \tan \pi x + \frac{1}{\pi} \ln |\sec \pi x + \tan \pi x| + C$$

$$\int \sec^3 \pi x \, dx = \frac{1}{2\pi} \left[\sec \pi x \tan \pi x + \ln |\sec \pi x + \tan \pi x| \right] + C$$

(31) $\int \tan^5 \frac{x}{4}\,dx$

$= \int \tan^3 \frac{x}{4}\left(\sec^2 \frac{x}{4} - 1\right)dx$

$= \int \tan^3 \frac{x}{4} \sec^2 \frac{x}{4}\,dx - \int \tan^3 \frac{x}{4}\,dx$

$u = \tan \frac{x}{4}$

$- \int \tan \frac{x}{4}\left(\sec^2 \frac{x}{4} - 1\right)dx$

$du = \frac{1}{4} \sec^2 \frac{x}{4}\,dx$

$- \int \tan \frac{x}{4} \sec^2 \frac{x}{4}\,dx + \int \tan \frac{x}{4}\,dx$

$= \int 4u^3\,du - \int 4u\,du - 4 \ln\left|\cos \frac{x}{4}\right| + C$

$= \tan^4 \frac{x}{4} - 2\tan^2 \frac{x}{4} - 4 \ln\left|\cos \frac{x}{4}\right| + C$

35 $\int \tan^2 x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int u^2 \, du = \frac{1}{3} u^3 + C$$

$$= \frac{1}{3} \tan^3 x + C$$

37 $\int \sec^6 4x \tan 4x \, dx$

$u = \sec 4x$

$du = 4 \sec 4x \tan 4x \, dx$

$= \frac{1}{4} \int u^5 \, du = \frac{1}{24} u^6 + C$

$= \frac{1}{24} \sec^6 4x + C$

41 $\displaystyle\int \frac{\tan^2 x}{\sec x}\,dx$

$$= \int \frac{\sec^2 x - 1}{\sec x}\,dx$$

$$= \int (\sec x - \cos x)\,dx$$

$$= \ln|\sec x + \tan x| - \sin x + C$$

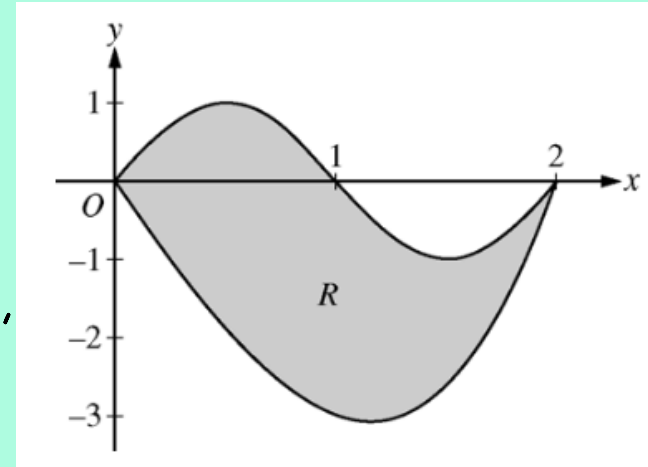
8.3b Let's Review everything!!

At the end of this lesson you will be able to:

- Successfully solve every type of problem in this unit!

BC 2008 #1

Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure to the right.



a) Find the area of R.

b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

2003 MC

15) The length of a curve from $x=1$ to $x=4$ is given by $\int_1^4 \sqrt{1+9x^4}\,dx$. If the curve contains the point $(1,6)$, which of the following could be an equation for this curve?

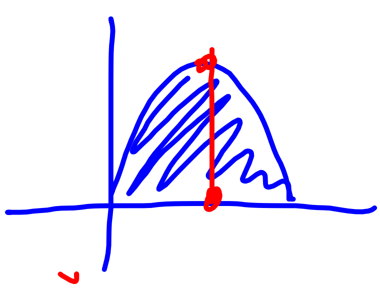
(A) $y=3+3x^2$
(B) $y=5+x^3$
(C) $y=6+x^3$
(D) $y=6-x^3$
(E) $y=\frac{16}{5}+x+\frac{9}{5}x^5$

23) $\int x\sin(6x)\,dx =$

(A) $-x\cos(6x)+\sin(6x)+C$
(B) $-\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$
(C) $-\frac{x}{6}\cos(6x)+\frac{1}{6}\sin(6x)+C$
(D) $\frac{x}{6}\cos(6x)+\frac{1}{36}\sin(6x)+C$
(E) $6x\cos(6x)-\sin(6x)+C$

89) The region bounded by the graph of $y=2x-x^2$ and the x-axis is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is an equilateral triangle. What is the volume of this solid?

(A) 1.333
(B) 1.067
(C) 0.577
(D) 0.462
(E) 0.267



Self-practice! Online is a set of 10 practice tests that cover all of the integrals we have learned so far. (There might be a random problem here or there that requires partial fractions to solve, so feel free to guess on these.) Continue taking quizzes until you can get a perfect score.

Get to these by going to my website and clicking on the 'practice quizzes online for integration' link (or type in the website below).

http://cims.nyu.edu/~kiryl/Online%20Tests/Set_65.html

What have we learned?

- Can I successfully solve any problem from this unit?

January 20, 2016