Let's warm up with a warmup!!

Assume the volume of a cube is increasing at a constant rate of 3 cubic cm per second. Let t_0 be the instant when the rate of change of the volume is equal to the rate of change of the surface area for the cube. Assume v = 0 when t = 0. (Hint: solve for the x-values, then solve for dx/dt, then separate and integrate to solve for t)

a) Find the value(s) of t_0

b) Find the rate of change of a side when $t = t_0$

c) Find the rate of change of the surface area when $t = t_0$

✓a) $t_0 = 64$ b) $dx/dt = 1/16$ c) $dS/dt = 3$

13

$$\int x^3 e^x\,dx = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$$

	u	dv
$+$	x^3	e^x
$-$	$3x^2$	e^x
$+$	$6x$	e^x
$-$	6	e^x
$+$	0	e^x

17 $\int t \ln(t+1)\, dt$

$u = \ln(t+1) \qquad du = \frac{1}{t+1}\, dt$

$v = \frac{1}{2} t^2 \qquad dv = t\, dt$

$= \frac{1}{2} t^2 \ln(t+1) - \int \frac{1}{2} \cdot \frac{t^2}{t+1}\, dt$

$$\begin{array}{r|l} & t - 1 \\ \hline t+1 & t^2 + 0t + 0 \\ & t^2 + t \\ \hline & -t + 0 \\ & -t - 1 \\ \hline & 1 \end{array}$$

$= \frac{1}{2} t^2 \ln(t+1) - \frac{1}{2} \int \left(t - 1 + \frac{1}{t+1} \right) dt$

$= \frac{1}{2} t^2 \ln(t+1) - \frac{1}{4} t^2 + \frac{1}{2} t - \frac{1}{2} \ln|t+1| + C$

(19) $\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \frac{1}{3}(\ln x)^3 + C$

21

$$\int \frac{xe^{2x}}{(2x+1)^2}dx$$

let $u = xe^{2x}$ so $du = e^{2x} + 2xe^{2x}$
let $dv = (2x+1)^{-2}$ so $v = -\frac{1}{2}(2x+1)^{-1}$

$$= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{e^{2x} + 2xe^{2x}}{2(2x+1)}dx$$

$$= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{e^{2x}(1+2x)}{2(2x+1)}dx$$

$$= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{1}{2}e^{2x}dx$$

$$= -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C$$

recognizing that you can factor out the 'e' term and cancel common factors was the key to this problem

27 $\int x\cos x\,dx$

	u	dv
$+$	x	$\cos x$
$-$	1	$\sin x$
$+$	0	$-\cos x$

$= x\sin x + \cos x + C$

(31) $\int t\csc t\cot t\,dt$

$u = t \qquad du = dt$

$v = -\csc t \qquad dv = \csc t\cot t$

$= -t\csc t + \int \csc t\,dt$

$= -t\csc t - \ln|\csc t + \cot t| + C$

35 $\int e^{2x}\sin x\,dx$

$u=\sin x \quad du=\cos x\,dx$

$v=\frac{1}{2}e^{2x} \quad dv=e^{2x}\,dx$

$=\frac{1}{2}e^{2x}\sin x-\int\frac{1}{2}e^{2x}\cos x\,dx$

$u=\cos x \quad du=-\sin x$

$v=\frac{1}{2}e^{2x} \quad dv=e^{2x}\,dx$

$$\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{2}\left[\frac{1}{2}e^{2x}\cos x+\int\frac{1}{2}e^{2x}\sin x\,dx\right]$$

$$\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x-\frac{1}{4}\int e^{2x}\sin x\,dx$$

$$\frac{5}{4}\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x+C$$

$$\int e^{2x}\sin x\,dx=\frac{2}{5}e^{2x}\sin x-\frac{1}{5}e^{2x}\cos x+C$$

$$\int \underbrace{x\ln x}_{u}\,\underbrace{dx}_{dv} = uv - \int v\,du$$

(so $v = x$)

$$\int x\ln x\,dx = x^2\ln x - \int x\cdot d(x\ln x)\,dx$$

$$= x^2\ln x - \int x(\ln x + 1)\,dx$$

$$= x^2\ln x - \int (x\ln x + x)\,dx$$

$$\underline{\int x\ln x\,dx} = x^2\ln x - \underline{\int x\ln x\,dx} - \int x\,dx$$

$$2\int x\ln x\,dx = x^2\ln x - \frac{1}{2}x^2$$

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

8.2b More Integration by Parts!!

At the end of this lesson you will be able to:

- Become more confident in knowing which type of problems require IBP and feel like an expert at using the method

Which of the following integrals requires integration by parts? Discuss with your group and make a decision. Then integrate all three.

1) $\int x^2 e^{3x} dx$

2) $\int \frac{x}{\sqrt{2+3x}} dx$

3) $\int \frac{\ln(2x)}{x^2} dx$

1) $\int x^2 e^{3x} dx$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$

u	dv
$+x^2$	e^{3x}
$-2x$	$1/3e^{3x}$
$+2$	$1/9e^{3x}$
-0	$1/27e^{3x}$

2) $\int \frac{x}{\sqrt{2+3x}} dx$

$$= \int \frac{1}{3} \cdot \frac{\frac{1}{3}(u-2)}{\sqrt{u}} du$$

$$= \int \frac{1}{9}\left(u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\right) du$$

$$= \frac{2}{27}(2+3x)^{\frac{3}{2}} - \frac{4}{9}(2+3x)^{\frac{1}{2}} + C$$

3) $\int \frac{\ln(2x)}{x^2} dx$

$$= -\frac{\ln(2x)}{x} - \int -\frac{1}{x^2} dx$$

$$= -\frac{\ln(2x)}{x} - \frac{1}{x} + C$$

$u = \ln(2x)$ $du = 1/x\ dx$

$v = -x^{-1}$ $dv = x^{-2}\ dx$

Say, it's been a while since we've worked on definite integrals! Let's give some a try that require integration by parts!

Best method: evaluate the entire integral, make sure it's all in terms of x, and then evaluate it for the given limits

$$ex) \int_0^{\pi} x \sin(2x)\, dx =$$

	u	dv
$+$	x	$\sin(2x)$
$-$	1	$-\frac{1}{2}\cos(2x)$
$+$	0	$-\frac{1}{4}\sin(2x)$

✓ $$= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x)\Big|_0^{\pi} = -\frac{\pi}{2}$$

You try! Evaluate the following:

1) $\int_0^1 \ln(1+x^2)\,dx = \ln 2 - 2 + \frac{\pi}{2}$ ✓

$u = \ln(1+x^2) \quad du = \frac{2x}{1+x^2}$

$v = x \quad dv = dx$

$= x\ln(1+x^2)\Big|_0^1 - \underbrace{\int_0^1 \frac{2x^2}{1+x^2}\,dx}$

$$\begin{array}{r} 2 \\ x^2+1 \overline{\big)\, 2x^2+0} \\ -(2x^2+2) \\ \hline -2 \end{array}$$

$= \int_0^1 2 - \frac{2}{x^2+1}\,dx$

$= x\ln(1+x^2) - 2x + 2\arctan x \Big|_0^1$

$= \ln 2 - 2 + 2\cdot\frac{\pi}{4} = \ln 2 - 2 + \frac{\pi}{2}$

2) $\int_0^1 x \arcsin x^2\,dx = \frac{\pi}{4} - \frac{1}{2}$ ✓

3) $\int_0^1 x^3 e^{-2x} dx = \frac{3e^2 - 19}{8e^2}$ ✓

2015 BC exam #4

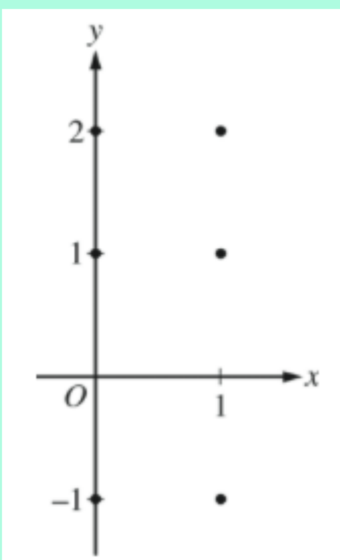
Consider the differential equation dy/dx = 2x - y

a) On the axes provided, sketch a slope field for the differential equation at the six points indicated

b) Find d^2y/dx^2 in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

c) Let y = f(x) be the particular solution to the differential equation with the initial condition f(2) = 3. Does f have a relative minimum, a relative maximum, or neither at x = 2? Justify your answer.

d) Find the values of the constants m and b for which y = mx + b is a solution to the differential equation.



2015 BC #1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin(t^2 / 35)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

What have we learned?

- Can I use integration by parts to evaluate integrals?
- Can I determine when integration by parts is necessary?
- Can I use tab method if it applies?