Let's warm up with a warmup!!

Challenge: Evaluate $\int xe^x dx$

We know $d/dx\ xe^x = e^x + xe^x$

✓ hmmm... let's try $d/dx\ xe^x - e^x = e^x + xe^x - e^x = xe^x$!!

So $\int xe^x dx = xe^x - e^x + C$

Wouldn't it be great if there was a way to figure this out without using guess and check?

23 $\displaystyle\int \frac{x^2}{x-1}\,dx = \int \frac{x^2-1}{x-1}\,dx + \int \frac{1}{x-1}\,dx$

$$\begin{array}{r|rrr} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

$$= \int x + 1 + \frac{1}{x-1}\,dx$$

$$= \frac{1}{2}x^2 + x + \ln|x-1| + C$$

39 $\displaystyle\int \frac{1+\sin x}{\cos x}\,dx$

$$= \int (\sec x + \tan x)\,dx$$

$$= \ln|\sec x + \tan x| - \ln|\cos x| + C$$

$$= \ln \frac{|\sec x + \tan x|}{|\cos x|} + C$$

$$= \ln|\sec^2 x + \sec x \tan x| + C$$

41 $\displaystyle\int \frac{1}{\cos\theta - 1}\,d\theta$ multiply by $\cos\theta + 1$

$$= \int \frac{\cos\theta + 1}{(\cos^2\theta - 1)}\,d\theta$$

$$= \int \frac{\cos\theta + 1}{-\sin^2\theta}\,d\theta$$

$$= \int \left(-\csc\theta\cot\theta - \csc^2\theta\right) d\theta$$

$$= \csc\theta + \cot\theta + C$$

43 $\int \frac{-1}{\sqrt{1-(2t-1)^2}}\,dt$

$= \frac{-1}{2}\arcsin(2t-1) + C$

$\int \sin(2t)\,dt = -\frac{1}{2}\cos(2t) + C$

$\int e^{2t}\,dt = \frac{1}{2}e^{2t} + C$

$\ddots$

45 $\displaystyle\int \frac{\tan\left(\frac{2}{t}\right)}{t^2}\,dt$

$u = \frac{2}{t}$ $\qquad = \displaystyle\int -\frac{1}{2}\tan u\,du$

$du = -\frac{2}{t^2}\,dt$

$-\frac{1}{2}du = \frac{1}{t^2}\,dt$ $\qquad = \frac{1}{2}\ln|\cos u| + C$

$= \frac{1}{2}\ln\left|\cos\left(\frac{2}{t}\right)\right| + C$

47 $\int \frac{3}{\sqrt{6x-x^2}}\,dx$

$$= \int \frac{3}{\sqrt{-(x^2-6x+9)+9}}\,dx$$

$$= \int \frac{3}{\sqrt{9-(x-3)^2}}\,dx$$

$$= 3\arcsin\frac{x-3}{3} + C$$

49

$$\int \frac{4}{4x^2+4x+65}\,dx$$

$$= \int \frac{4}{4\left(x^2+x+\frac{1}{4}\right)+65-1}\,dx$$

$$= \int \frac{4}{4\left(x+\frac{1}{2}\right)^2+64}\,dx$$

$$= \int \frac{1}{\left(x+\frac{1}{2}\right)^2+16}\,dx$$

$$= \frac{1}{4}\arctan\frac{x+\frac{1}{2}}{4} + C$$

8.2a Integration by Parts!!

Essential Learning Target:

- Techniques for finding antiderivatives include algebraic manipulation such as long division, completing the square, substitution of variables and integration by parts

Let's derive the method!!

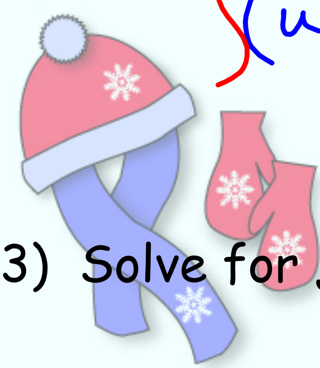
1) Write out product rule:

$$(f \cdot g)' = f'g + fg'$$

2) Write it out again using u and v instead of f and g

$$\int (uv)' = \int u'v + \int uv'$$

$$uv = \int v\,du + \int u\,dv$$



3) Solve for $\int udv$

$$\int u\,dv = uv - \int v\,du$$

Integration by Parts: $\int udv = uv - \int vdu$

LIPET

L = logarithms

I = inverse trig

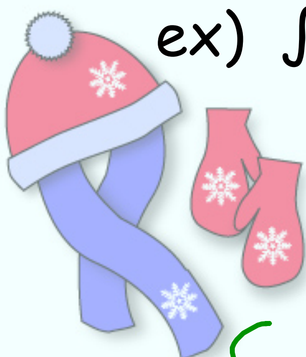
P = polynomials

E = exponentials

T = trig

When deciding on u and dv, use LIPET to choose your u.

Then dv is whatever is left.



ex) $\int \underline{x}\,\underline{e^x dx}$

$u = x \xrightarrow{\text{differentiate}} du = dx$

$v = e^x \xleftarrow{\text{antidifferentiate}} dv = e^x dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^x dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

You try!

1) $\int x\sec x\tan x\,dx$

$u = x \qquad du = dx$

$v = \sec x \qquad dv = \sec x\tan x\,dx$

$\int u\,dv = uv - \int v\,du$

$\int x\sec x\tan x\,dx$

$= x\sec x - \int \sec x\,dx$

$= x\sec x - \ln|\sec x + \tan x| + C$

ex) $\int \ln x dx =$

$u = \ln x \quad du = \frac{1}{x} dx$

$v = x \quad dv = dx$

$\int \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - \int dx$

$= x \ln x - x + C$

ex) $\int x \ln x dx =$

$u = \ln x \quad du = \frac{1}{x} dx$

$v = \frac{1}{2}x^2 \quad dv = x dx$

$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx$

$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

Tabular Method! You can use this anytime your function is the product of a polynomial and any other function that you can <u>repeatedly</u> integrate.

ex) $\int x^4 \sin x\,dx$

	u	dv
+	x^4	$\sin x$
−	$4x^3$	$-\cos x$
+	$12x^2$	$-\sin x$
−	$24x$	$\cos x$
+	24	$\sin x$
−	0	$-\cos x$

$$\int x^4 \sin x\,dx = -x^4\cos x + 4x^3\sin x + 12x^2\cos x - 24x\sin x - 24\cos x + C$$

You try) $\int (x^2 - 3x)e^x dx$

	u	dv
+	x^2-3x	e^x
−	$2x-3$	e^x
+	2	e^x
−	0	e^x

$$\int (x^2-3x)e^x dx = (x^2-3x)e^x - (2x-3)e^x + 2e^x + C$$

Slight challenge - let's grow those brains!

ex) $\int e^x \cos x\,dx$

$u = e^x \qquad du = e^x\,dx$

$v = \sin x \qquad dv = \cos x\,dx$

$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$

$u = e^x \qquad du = e^x\,dx$

$v = -\cos x \qquad dv = \sin x\,dx$

$\int e^x \cos x\,dx = e^x \sin x - \left[-e^x \cos x - \int -e^x \cos x\,dx\right]$

$\underline{\int e^x \cos x\,dx} = e^x \sin x + e^x \cos x - \underline{\int e^x \cos x\,dx}$

$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x\,dx = \frac{1}{2}\left[e^x \sin x + e^x \cos x\right] + C$

1) Evaluate $\int \ln(2 + 3x^2)dx$

2) Rotate the region bounded by the curves $y = 2\sqrt{x}$, $y = -\frac{1}{2}x + 6$ and $y = 0$ about the line $y = -1$

a) What is the volume of the solid formed?

b) What is the perimeter of the region?

c) What is the TOTAL surface area of the solid?

1) ✓

$$\int \ln(2+3x^2)\,dx = x\ln(2+3x^2) - \int \frac{6x^2}{2+3x^2}\,dx$$

$$= x\ln(2+3x^2) - 2x + \int \frac{4}{2+3x^2}\,dx$$

$$= x\ln(2+3x^2) - 2x + \frac{4\sqrt{3}}{3\sqrt{2}}\arctan\frac{x\sqrt{3}}{\sqrt{2}} + C$$

2a) ✓

$$V = \pi\int_0^4 \left[\left(2\sqrt{x}+1\right)^2 - (0+1)^2\right]dx + \pi\int_4^{12}\left[\left(-\frac{1}{2}x+6+1\right)^2 - (0+1)^2\right]dx$$

$\approx 53.333\pi + 74.666\pi = 128\pi \approx 402.123$ or 402.124

OR

$$V = 2\pi\int_0^4 (y+1)\left(-2y+12-\frac{y^2}{4}\right)dy = 128\pi \approx 402.123\ or\ 402.124$$

2b) ✓

$$S = \int_0^4 \sqrt{1+\left(x^{-\frac{1}{2}}\right)^2}\,dx + \int_4^{12}\sqrt{1+\left(-\tfrac{1}{2}\right)^2}\,dx + 12$$

$$\approx 5.915 + 8.944 + 12 \approx 26.860$$

2c) ✓

$$SA = 2\pi\int_0^4 \left(2\sqrt{x}+1\right)\sqrt{1+\left(x^{-\frac{1}{2}}\right)^2}\,dx +$$

$$2\pi\int_4^{12}\left(-\tfrac{1}{2}x+6+1\right)\sqrt{1+\left(-\tfrac{1}{2}\right)^2}\,dx +$$

$$2\pi\int_0^{12}(1)\sqrt{1+0}\,dx$$

$\approx 38.979\pi + 53.665\pi + 24\pi \approx 116.644\pi$ or 116.645π or 366.450

All the way with PVA! (calculators permitted)

A particle moves along the x-axis so that its velocity is given by $v(t) = 8\sin(t) - \frac{1}{4}e^{\frac{t}{2}} + t - 2$

for all t, $1 \le t \le 4$.

When t = 1, the position of the particle is x(1) = 11.

a) What is the particle's acceleration at t = 2?

b) At what time(s), $1 < t < 4$ does the acceleration equal 0?

c) At what time(s) does the particle change direction?

d) What is the total distance traveled by the particle on the interval $1 \le t \le 4$?

e) What is the particle's displacement on $1 \le t \le 4$?

f) What is the particle's position at t = 4?

a) $a(2) = v'(2) \approx -2.668$ or -2.669

b) $a(t) = 0$ at $t \approx 1.660$

c) changes direction at $t \approx 3.133$ (or 3.134) because v(t) changes sign

d) distance = $\int |v(t)| dt \approx 13.605$

e) displacement = $\int v(t) dt \approx 8.181$

f) $\int_1^4 v(t)dt = x(4) - x(1)$ so $x(4) \approx 8.181 + 11 = 19.181$

What have we learned?

- Can I use integration by parts to evaluate integrals?
- Can I determine when integration by parts is necessary?
- Can I use tab method if it applies?

January 12, 2017