Let's warm up with a warmup!!

2009 #5 (no calculators)

x	2	3	5	8	13
f(x)	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

a) Estimate $f'(4)$. Show the work that leads to your answer.

b) Evaluate $\int_2^{13}(3 - 5f'(x))dx$. Show the work that leads to your answer.

c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13}(3 - 5f'(x))dx$ Show the work that leads to your answer.

d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge 4/3$.

(1) $y = \frac{1}{10}x^5 + \frac{1}{6}x^{-3}$ $[1, 2]$

$y' = \frac{1}{2}x^4 - \frac{1}{2}x^{-4}$

$$AL = \int_1^2 \sqrt{1 + \left(\frac{x^4}{2} - \frac{1}{2x^4}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \left(\frac{x^8 - 1}{2x^4}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{x^{16} - 2x^8 + 1}{4x^8}}\, dx$$

$$= \int_1^2 \sqrt{\frac{x^{16} + 2x^8 + 1}{4x^8}}\, dx$$

$$= \int_1^2 \sqrt{\frac{(x^8+1)^2}{(2x^4)^2}}\, dx = \int_1^2 \frac{x^8 + 1}{2x^4}\, dx$$

$$= \int_1^2 \left(\frac{1}{2}x^4 + \frac{1}{2}x^{-4}\right) dx$$

$$= \frac{1}{10}x^5 - \frac{1}{6}x^{-3} \Big|_1^2$$

$$= \frac{32}{10} - \frac{1}{6}\cdot\frac{1}{8} - \left(\frac{1}{10} - \frac{1}{6}\right)$$

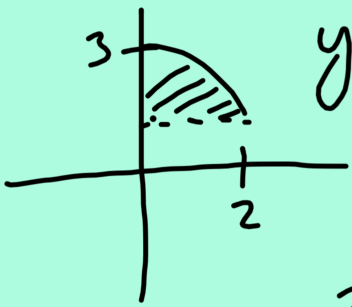
$$= \frac{16}{5} - \frac{1}{48} - \frac{1}{10} + \frac{1}{6} = \boxed{\frac{779}{240}}$$

(41) $y = \frac{1}{6}x^3 + \frac{1}{2x}$ $[1, 2]$

$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$

$$SA = 2\pi \int_1^2 \underbrace{\left(\frac{1}{6}x^3 + \frac{1}{2x}\right)}_{r} \underbrace{\sqrt{1 + \left(\frac{1}{2}x^2 - \frac{1}{2x^2}\right)^2}}_{AL}\, dx$$

(53) $y = \sqrt{9 - x^2}$ $[0, 2]$



$y' = \frac{1}{2}(9 - x^2)^{-\frac{1}{2}}(-2x)$

$= \frac{-x}{\sqrt{9 - x^2}}$

$$SA = 2\pi \int_0^2 x \sqrt{1 + \left(\frac{-x}{\sqrt{9-x^2}}\right)^2}\, dx$$

(47) A rectifiable curve is a curve that has a finite arc length.

(49) Formula used for surface area is the surface area of a frustrum of a cone.

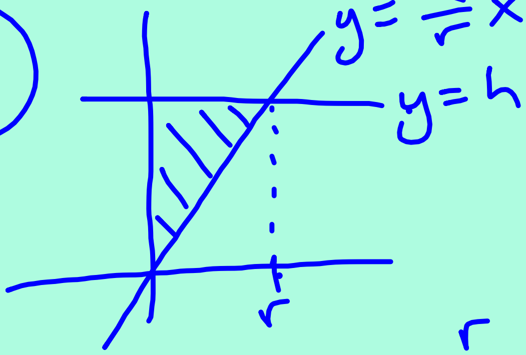
(23) $y = 2\arctan x \quad [0, 1]$

$$y' = 2 \cdot \frac{1}{1+x^2}$$

$$(y')^2 = \frac{4}{(1+x^2)^2}$$

$$AL = \int_0^1 \sqrt{1 + \frac{4}{(1+x^2)^2}}\, dx$$

(51)



$y = \frac{h}{r}x \rightarrow$ so $y' = \frac{h}{r}$

$$SA = 2\pi \int_0^r x\sqrt{1+\left(\frac{h}{r}\right)^2}\,dx$$

$$= 2\pi \int_0^r x\sqrt{\frac{r^2+h^2}{r^2}}\,dx$$

$$= 2\pi \cdot \frac{1}{2}x^2 \sqrt{\frac{r^2+h^2}{r^2}}\Big|_0^r$$

$$= \pi r^2 \cdot \frac{\sqrt{r^2+h^2}}{r} = \pi r\sqrt{r^2+h^2}$$

8.1 Integration Review!!

At the end of this lesson you will be able to:

- Effectively determine which method to use to approach any integral that is thrown your way

2nd Warmup! Evaluate each of the following:

a) $\int \frac{4}{x^2+9}\,dx$ b) $\int \frac{4x}{x^2+9}\,dx$ c) $\int \frac{4x^2}{x^2+9}\,dx$

✓ $a) \int \frac{4}{x^2+9}\,dx = \frac{4}{3}\arctan\frac{x}{3} + C$

✓ $b) \int \frac{4x}{x^2+9}\,dx = \int 2 \cdot \frac{1}{u}\,du = 2\ln(x^2+9) + C$

✓ $c) \int \frac{4x^2}{x^2+9}\,dx = \int \left(4 - \frac{36}{x^2+9}\right)dx = 4x - 12\arctan\frac{x}{3} + C$

Page 520 in your textbook has a nice overview of the integrals you have learned so far.

Page 521 has a nice list of procedures (shown below) you can use if the basic integral rules, u-substitution and division are not cutting it.

1) Expand a function $(1+e^x)^2 = 1 + 2e^x + e^{2x}$
2) Separate the numerator $\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$
3) Complete the square $\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$
4) Long division $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$
5) Add and subtract terms in numerator $\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1}$
6) Use trigonometric identities $\cot^2 x = \csc^2 x - 1$
7) Multiply and divide by Pythagorean conjugate

$$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$$

Try the following!

✓ 1) $\int \frac{4}{e^{-3x}+1}dx = \int \frac{4}{\frac{1}{e^{3x}}+1}dx = \int \frac{4e^{3x}}{1+e^{3x}}dx = \int \frac{4}{3}\cdot\frac{1}{u}du = \frac{4}{3}\ln(1+e^{3x}) + C$

✓ 2) $\int \frac{3}{\sqrt{25-4x^2}}dx = \int \frac{3}{2}\cdot\frac{1}{\sqrt{\frac{25}{4}-x^2}}dx = \frac{3}{2}\arcsin\frac{2x}{5} + C$

$\sqrt{25-(2x)^2}$

✓ 3) $\int (2x^2)(5^{3x^3})dx = \int \frac{2}{9}\cdot 5^u du = \frac{2}{9}\cdot\frac{5^{3x^3}}{\ln 5} + C$

✓ 4) $\int \frac{3x^2}{x+1}dx = \int \left(3x - 3 + \frac{3}{x+1}\right)dx = \frac{3}{2}x^2 - 3x + 3\ln|x+1| + C$

✓ 5) $\int \frac{3+\tan x}{\cos x}dx = \int (3\sec x + \sec x\tan x)dx = 3\ln|\sec x + \tan x| + \sec x + C$

BONUS!!!

Find the point of intersection of the two lines tangent to $\sqrt{x} - 3xy + \sqrt{y} = -9$ at $x = 1$ and $x = 4$.

(Calculators permitted, but answer must be exact) (Time limit, 10 minutes)

✓ (30/19, 30/19)

A few more to try!

✓ 1) $\int 3\cos(5x)\,dx = \int \frac{3}{5}\cos u\,du = \frac{3}{5}\sin(5x) + C$

✓ 2) $\int \frac{3}{2t^2+1}dt = \frac{3}{2}\int \frac{1}{t^2+\frac{1}{2}}dt = \frac{3\sqrt{2}}{2}\arctan\left(t\sqrt{2}\right) + C$

✓ 3) $\int \frac{3x^2}{1+x^2}dx = \int \left(3 - \frac{3}{x^2+1}\right)dx = 3x - 3\arctan x + C$

✓ 4) $\int \frac{-1}{3+e^{-t}}dt = \int \frac{-e^t}{3e^t+1}dt = \int -\frac{1}{3}\cdot\frac{1}{u}du = -\frac{1}{3}\ln(3e^t+1) + C$

✓ 5) $\int \frac{\csc x + \tan x}{\tan x}dx = \int (\csc x \cot x + 1)dx = -\csc x + x + C$

✓ 6) $\int \frac{1}{\sqrt{-x^2-2x+4}}dx = \int \frac{1}{\sqrt{5-(x+1)^2}}dx = \arcsin\frac{x+1}{\sqrt{5}} + C$

What have we learned?

- Can I evaluate a large variety of integrals?