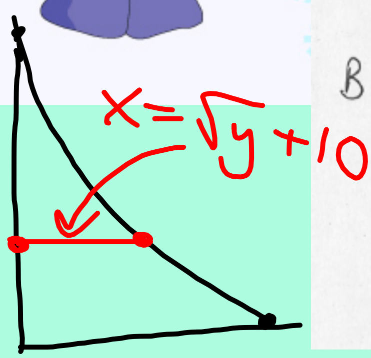
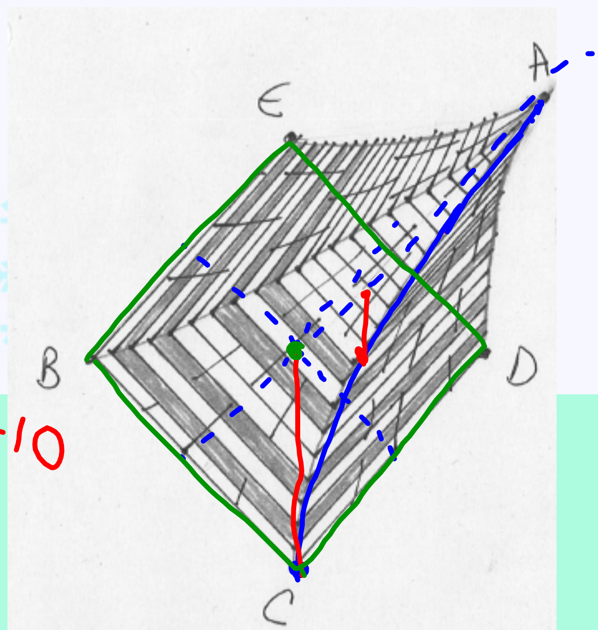
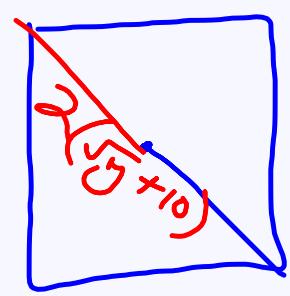
Let's warm up with a warmup!!

Imagine a structure that looks like the Eiffel Tower, except it's 'solid'. It would have a square base and four sides determined by four 'edges'. Imagine too that the center of the base is at the origin of a coordinate system and one 'edge' follows the curve $y = (x - 10)^2$ over [0, 10]. Find the volume of the structure.

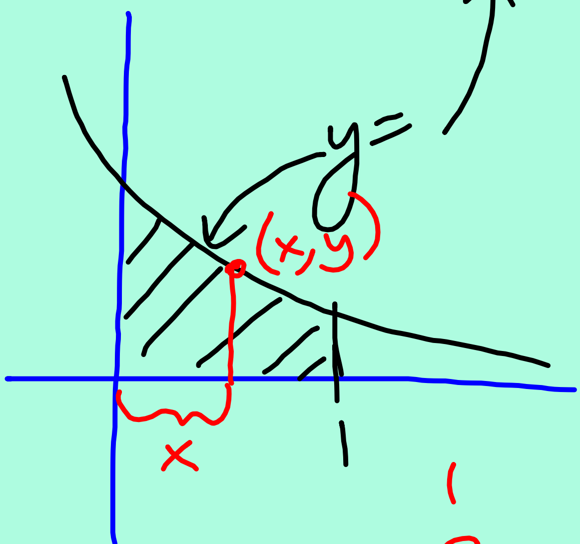






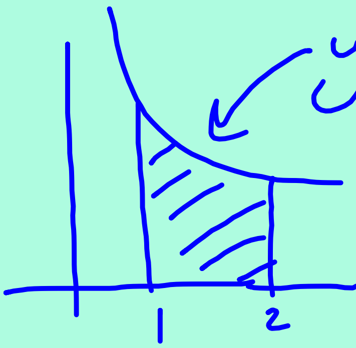
✓ $$V = \int_0^{100} 2(\sqrt{y} + 10)^2 dy = \frac{170000}{3} \approx 56666.666 \text{ or } 56666.667$$

(11) $y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$, $y = 0$, $x = 0$, $x = 1$



$$V = 2\pi \int_0^1 x \left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{1}{2}x^2} \, dx$$

(15)



$y = \frac{1}{x}$, $x = \frac{1}{y}$

Shell:

$$V = 2\pi \int_0^{\frac{1}{2}} y(2-1)\,dy$$

$$+ 2\pi \int_{\frac{1}{2}}^{1} y\left(\frac{1}{y}-1\right)dy$$

Of course, we would use disk for this problem

(29) $x^{\frac{1}{2}} + y^{\frac{1}{2}} = a^{\frac{1}{2}}$, $x=0$, $y=0$

$\sqrt{y} = \sqrt{a} - \sqrt{x}$

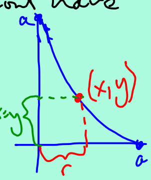
$y = (\sqrt{a} - \sqrt{x})^2$

x-int: $x^{\frac{1}{2}} = a^{\frac{1}{2}}$, $x = a$

a) disk:

$$V = \pi \int_0^a \left[(\sqrt{a} - \sqrt{x})^2 \right]^2 dx$$

shell (easier because you don't have to raise to 4^{th})



$$V = 2\pi \int_0^a y (\sqrt{a} - \sqrt{y})^2 \, dy$$

$$= 2\pi \int_0^a y (a - 2\sqrt{ay} + y) \, dy$$

$$= 2\pi \int_0^a (ay - 2\sqrt{a}\, y^{\frac{3}{2}} + y^2) \, dy$$

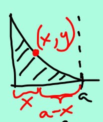
$$= 2\pi \left[\frac{1}{2} a y^2 - \frac{4}{5} a^{\frac{1}{2}} y^{\frac{5}{2}} + \frac{1}{3} y^3 \right] \Big|_0^a$$

$$= 2\pi \left[\frac{1}{2} a^3 - \frac{4}{5} a^3 + \frac{1}{3} a^3 \right]$$

$$= 2\pi \left(\frac{1}{30} a^3 \right) = \frac{\pi}{15} a^3$$

(b) same as a but with x^s

(c)



shell with dx

$$V = 2\pi \int_0^a (a - x) \left[\sqrt{a} - \sqrt{x} \right]^2 dx$$

$$= 2\pi \int_0^a (a - x) \left[a - 2\sqrt{ax} + x \right] dx$$

$$= 2\pi \int_0^a \left[a^2 - 2a\sqrt{ax} + ax - ax + 2x\sqrt{ax} - x^2 \right] dx$$

$$= 2\pi \int_0^a \left[a^2 - 2a^{\frac{3}{2}} x^{\frac{1}{2}} + 2a^{\frac{1}{2}} x^{\frac{3}{2}} - x^2 \right] dx$$

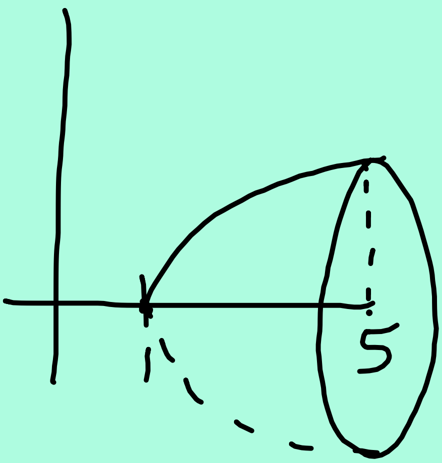
$$= 2\pi \left[a^2 x - \frac{4}{3} a^{\frac{3}{2}} x^{\frac{3}{2}} + \frac{4}{5} a^{\frac{1}{2}} x^{\frac{5}{2}} - \frac{1}{3} x^3 \Big|_0^a \right]$$

$$= 2\pi \left(a^3 - \frac{4}{3} a^3 + \frac{4}{5} a^3 - \frac{1}{3} a^3 \right]$$

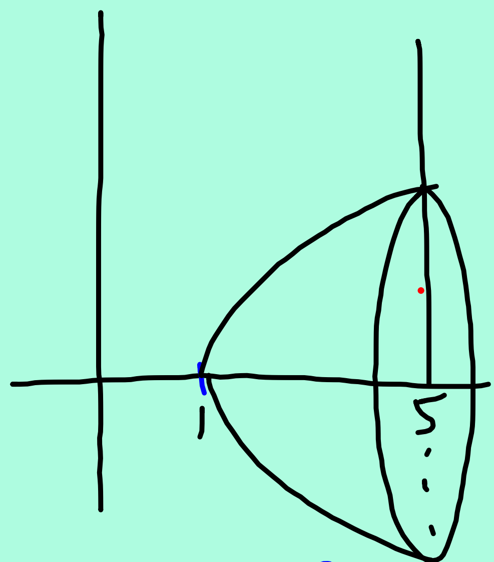
$$= \frac{4\pi}{15} a^3$$

(33)

$$\pi \int_1^5 (x-1)\,dx = 2\pi \int_0^2 y\left[5-(y^2+1)\right]dy$$



same shape,
same size



$x = y^2 + 1$

$y = \pm\sqrt{x-1}$

7.3b Putting it all together!!

At the end of this lesson you will be able to:

- Solve just about any problem dealing with area or volume :)

The Three Little Pigs!

(Taken from Wikipedia)
The Three Little Pigs was included in *The Nursery Rhymes of England* (1886), by James Orchard Halliwell-Phillipps. The story in its arguably best-known form appeared in *English Fairy Tales* by Joseph Jacobs, first published in 1890. The story begins with the title characters being sent out into the world by their mother, to "seek out their fortune". The first little pig builds a house of straw, but a wolf blows it down and eats him. The second pig builds a house of furze sticks, which the wolf also blows down and eats him. Each exchange between wolf and pig features ringing proverbial phrases, namely:

"Little pig, little Pig, let me come in."
"No, no, not by the hair on my chinny chin chin."
"Then I'll huff, and I'll puff, and I'll blow your house in."

The third pig builds a house of bricks. The wolf fails to blow down the house. He then attempts to trick the pig out of the house by asking to meet him at various places, but he is outwitted each time. Finally, the wolf resolves to come down the chimney, whereupon the pig catches the wolf in a cauldron of boiling water, slams the lid on, then cooks and eats him. (In another version, the pigs actually survive, each by running into the next pig's house.)

But, where are they now?

HOUSE A

Disk method:

$$V = \pi \int_0^{10} (7.5)^2 dy = \frac{1125}{2}\pi \approx 1767.145 \text{ or } 1767.146$$

Shell method:

$$V = 2\pi \int_0^{7.5} (x)(10) dx = \frac{1125}{2}\pi \approx 1767.145 \text{ or } 1767.146$$

Geometry:

$$V = \pi(7.5)^2(10) = \frac{1125}{2}\pi \approx 1767.145 \text{ or } 1767.146$$

HOUSE B

Cross Section method:

$$V = \int_0^{10} (15)(15) dx \text{ or } \int_0^{15} (15)(10) dx = 2250$$

Geometry:

$$V = (15)(15)(10) = 2250$$

HOUSE C

Disk Method:

$$V = \pi \int_0^{10} (10 - y)^2 dy = \frac{1000}{3}\pi \approx 1047.197 \text{ or } 1047.198$$

Shell method:

$$V = 2\pi \int_0^{10} (x)(10 - x) dx = \frac{1000}{3}\pi \approx 1047.197 \text{ or } 1047.198$$

Geometry:

$$V = \frac{1}{3}\pi(10)^2(10) = \frac{1000}{3}\pi \approx 1047.197 \text{ or } 1047.198$$

HOUSE D

Cross Section method:

$$V = \int_0^{10} (2(10 - x))^2 dx = \frac{4000}{3} \approx 1333.333$$

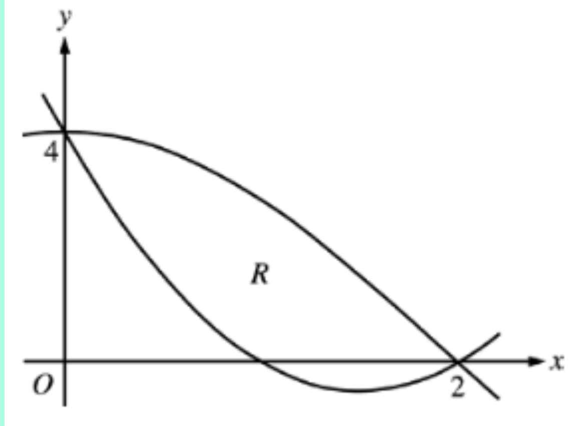
Geometry:

$$V = \frac{1}{3}(20)(20)(10) = \frac{4000}{3} \approx 1333.333$$

Let's take root with a kahoot!

(if time) 2013 #5) (no calc)

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos(\frac{1}{4}\pi x)$. Let R be the region bounded by the graphs of f and g, as shown in the figure.



a) Find the area of R.

b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

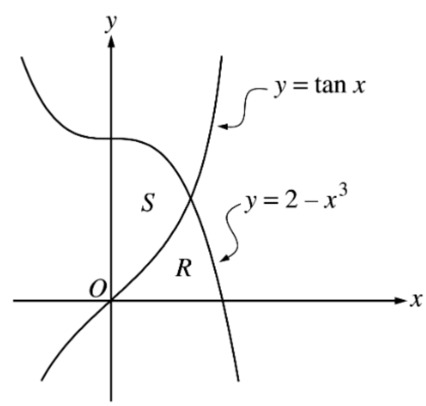
(a) $$\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$
$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$
$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

(b) $$\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$$
$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

(c) $$\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

(if time) 2001 #1 (calc)

Let R and S be the regions in the first quadrant shown in the figure. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and y = tanx. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and y = tanx.



a) Find the area of R
b) Find the area of S
c) Find the volume of the solid generated when S is revolved about the line y = 3
d) Find the volume of the solid generated when S is revolved about the line x = -2

Point of intersection

$2 - x^3 = \tan x$ at $(A, B) = (0.902155, 1.265751)$

(a) Area $R = \int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} (2 - x^3)\,dx = 0.729$

or

Area $R = \int_0^B \left((2-y)^{1/3} - \tan^{-1} y\right) dy = 0.729$

or

Area $R = \int_0^{\sqrt[3]{2}} (2 - x^3)\,dx - \int_0^A (2 - x^3 - \tan x)\,dx = 0.729$

(b) Area $S = \int_0^A (2 - x^3 - \tan x)\,dx = 1.160$ or 1.161

or

Area $S = \int_0^B \tan^{-1} y\,dy + \int_B^2 (2-y)^{1/3}\,dy = 1.160$ or 1.161

or

Area S

$= \int_0^2 (2-y)^{1/3}\,dy - \int_0^B \left((2-y)^{1/3} - \tan^{-1} y\right) dy$

$= 1.160$ or 1.161

c) $V = \pi \int_0^A (3 - \tan x)^2 - \left(3 - (2 - x^3)\right)^2 dx$

$= 4.311\pi$ or 13.543 or 13.544

d) $V = 2\pi \int_0^A (x + 2)(2 - x^3 - \tan x)\,dx$

$= 5.436\pi$ or 5.437π or 17.079 or 17.080

If there's even more time, Just for fun!

A particle moves along the x-axis such that its velocity, $v(t) = {}^{1}/_{2}\, t - 4\ln(t + 1) + 2$ on the interval $0 \le t \le 3$. When $t = 1$, the position of the particle is $x = 4$. Calculators are permitted, so use them!

a) When does the particle change direction?

b) What is the particle's acceleration when it changes direction?

c) What is the position of the particle at $t = 2$?

d) What is the total distance traveled by the particle?

a) $t \approx 0.828$ or 0.829

b) $a(t) \approx -1.687$

c) $x(2) \approx 3.111$ or 3.112

d) distance ≈ 3.387 or 3.388

What have we learned?

- Can I solve any problem related to area and volume?

January 08, 2016