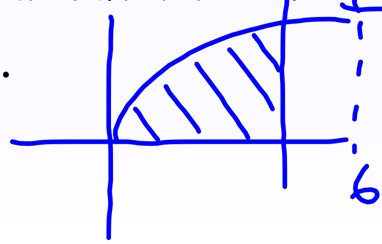
Let's warm up with a warmup!!

1) Use the trapezoid rule with n = 4 to approximate the area under the curve $y = 6 - x^2$ for $0 \le x \le 2$

2) Integrate $\displaystyle\int \frac{3}{6 + 2x^2}\,dx$

3) Solve $dy/dx = x^2 y$ for y, given $y(0) = 3$ and $y > 0$.

4) The region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$ is revolved about the line $x = 6$. Find the volume of the solid.



1) A = (1/2)(1/2)[(6 + 23/4) + (23/4 + 5) + (5 + 15/4) + (15/4 + 2)] = 37/4 or 9.25

2) $$\int \frac{3}{6 + 2x^2}\,dx = \frac{3}{2}\int \frac{1}{3 + x^2}\,dx = \frac{3}{2\sqrt{3}}\arctan\frac{x}{\sqrt{3}} + C$$

3)

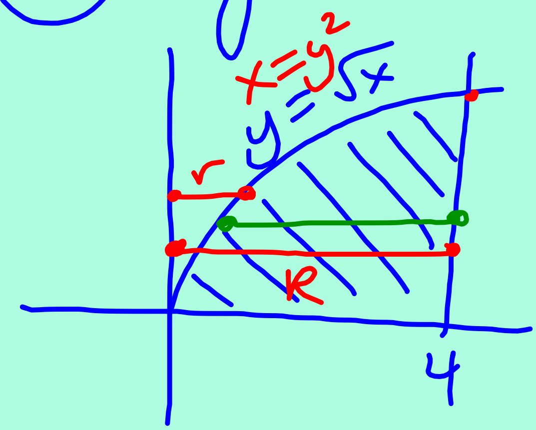
$$\int \frac{1}{y}\,dy = \int x^2\,dx \qquad \ln|y| = \tfrac{1}{3}x^3 + \ln 3$$

$$\ln|y| = \tfrac{1}{3}x^3 + C \qquad |y| = e^{\frac{1}{3}x^3 + \ln 3} = 3e^{\frac{1}{3}x^3}$$

$$C = \ln 3 \qquad y = 3e^{\frac{1}{3}x^3}$$

4) $$V = \pi\int_0^2 \left((6 - y^2)^2 - (6 - 4)^2\right)dy = \frac{192}{5}\pi \approx 120.637$$

11) $y=\sqrt{x}$, $y=0$, $x=4$



b) y-axis

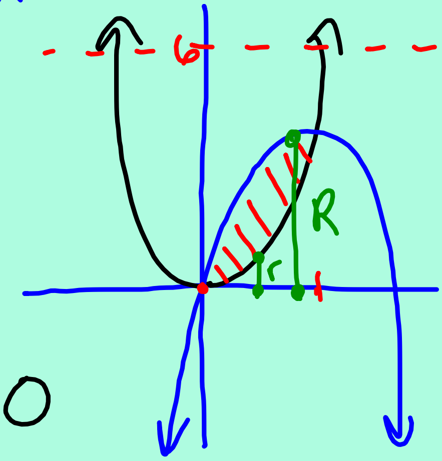
$$V=\pi\int_0^2\left[(4-0)^2-(y^2-0)^2\right]dy$$

c) $x=4$

$$V=\pi\int_0^2(4-y^2)^2\,dy$$

13) $y = x^2,\ y = 4x - x^2$

a) x-axis

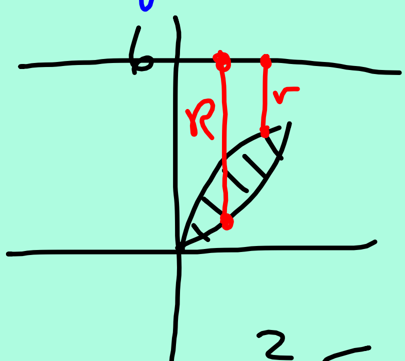


$$x^2 = 4x - x^2$$
$$2x^2 - 4x = 0$$
$$x^2 - 2x = 0$$
$$x = 0, 2$$

$$R = 4x - x^2 - 0$$
$$r = x^2 - 0$$

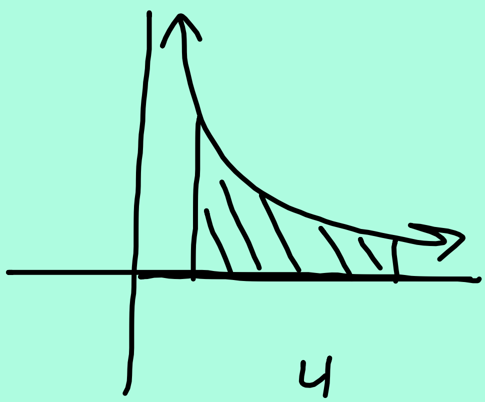
$$V = \pi \int_0^2 \left[\left(4x - x^2\right)^2 - \left(x^2 \right)^2 \right] dx$$

b) $y = 6$



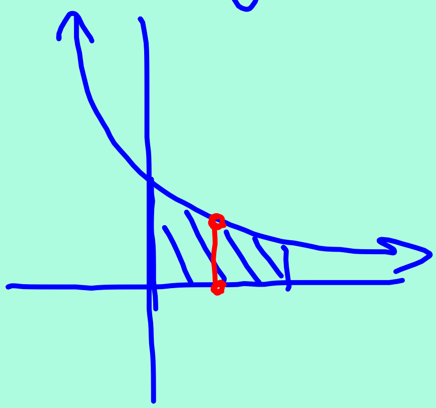
$$V = \pi \int_0^2 \left[\left(6 - x^2\right)^2 - \left(6 - (4x - x^2)\right)^2 \right] dx$$

25 $y=\frac{1}{x}$, $y=0$, $x=1$, $x=4$



$$V=\pi\int_1^4\left(\frac{1}{x}\right)^2dx$$

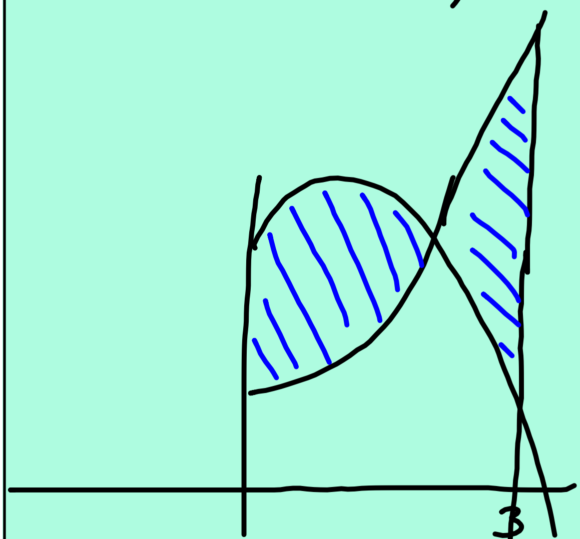
27 $y = e^{-x}$, $y = 0$, $x = 0$, $x = 1$



$$V = \pi \int_0^1 \left(e^{-x}\right)^2 dx$$

(29) $y = x^2 + 1$, $y = -x^2 + 2x + 5$

$x = 0$, $x = 3$



$x^2 + 1 = -x^2 + 2x + 5$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$$V = \pi \int_0^2 \left[(-x^2+2x+5)^2 - (x^2+1)^2 \right] dx$$

$$+ \pi \int_2^3 \left[(x^2+1)^2 - (-x^2+2x+5)^2 \right] dx$$

7.2b Volumes by Cross Sections!!

At the end of this lesson you will be able to:

- Calculate the volume of a solid using the cross section of the solid

What on earth do these shapes look like?

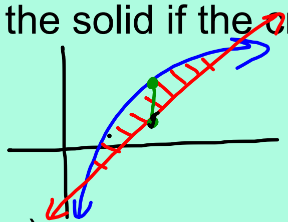
Let's see!

Volume by Cross Section!

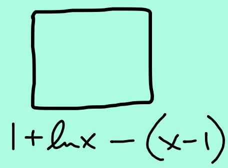
$$\text{Volume} = \int (\textit{area of cross section})\ (dx \textit{ or } dy)$$

cross sections are perpendicular to x-axis => dx

cross sections are perpendicular to y-axis => dy ex) The base of a solid is the region bounded by y = 1+ lnx and y = x - 1. Find the volume of the solid if the cross sections are:



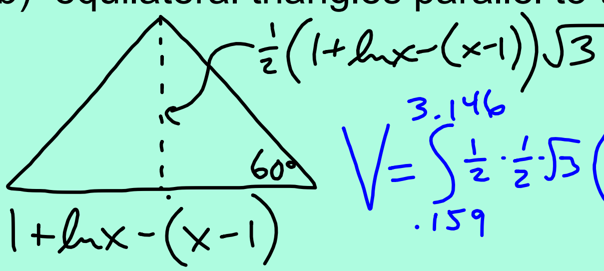
a) squares perpendicular to the x-axis



$$V=\int_{.159}^{3.146}\left[1+\ln x-(x-1)\right]^2 dx$$

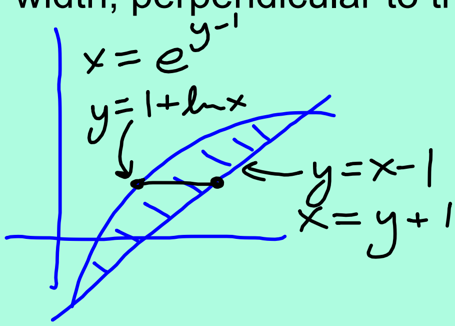
$$\approx 1.545$$

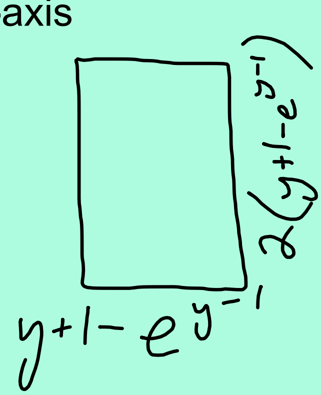
b) equilateral triangles parallel to the y-axis



$$V=\int_{.159}^{3.146}\frac{1}{2}\cdot\frac{1}{2}\cdot\sqrt{3}\left(1+\ln x-(x-1)\right)^2 dx$$

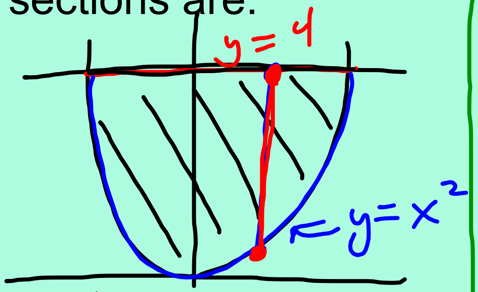
c) rectangles where each height is twice the width, perpendicular to the y-axis



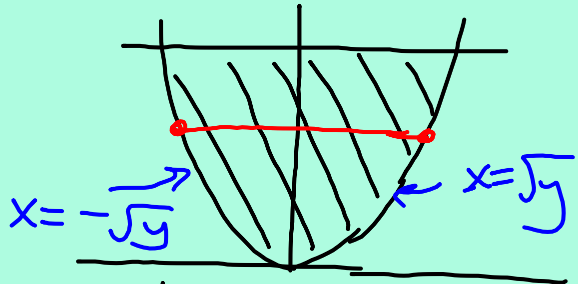


$$V=\int_{-0.841}^{2.146}2\left(y+1-e^{y-1}\right)^2 dy$$

You try! The base of a solid is bounded by $y = x^2$ and $y = 4$. Find the volume of the solid if the cross sections are:



$dx: 4 - x^2$



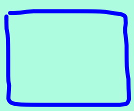
$dy: \sqrt{y} - -\sqrt{y}$
$= 2\sqrt{y}$

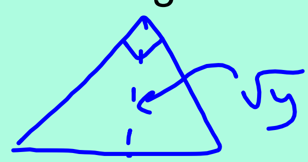
1) squares parallel to the x-axis

2) isosceles right triangles with the hypotenuse on the base perpendicular to the y-axis

3) semicircles parallel to the y-axis

4) isosceles right triangles with one leg on the base perpendicular to the x-axis





$$1)\ V = \int_0^4 \left(2\sqrt{y}\right)^2 dy = 32$$

$$2)\ V = \int_0^4 \frac{1}{2}\left(2\sqrt{y}\right)\left(\sqrt{y}\right) dy = 8$$

$$3)\ V = \pi \int_{-2}^{2} \frac{1}{2}\left(\frac{4 - x^2}{2}\right)^2 dx = \frac{64}{15}\pi \approx 13.404$$

$$4)\ V = \int_{-2}^{2} \frac{1}{2}(4 - x^2)^2 dx = \frac{256}{15} \approx 17.066 \text{ or } 17.067$$

What have we learned?

- Can I find the volume of a solid with consistent geometrical cross sections?