Let's warm up with a warmup!!

Evaluate the following:

1) $\int 3\cos(5x)\,dx$

2) $\int \frac{\ln(2x)}{x}\,dx$

3) $\int x e^{\pi x^2}\,dx$

4) $\int \frac{3}{2t^2+1}\,dt$

✓

1) $\frac{3}{5}\sin(5x)+C$

2) $\frac{1}{2}\ln^2(2x)+C$

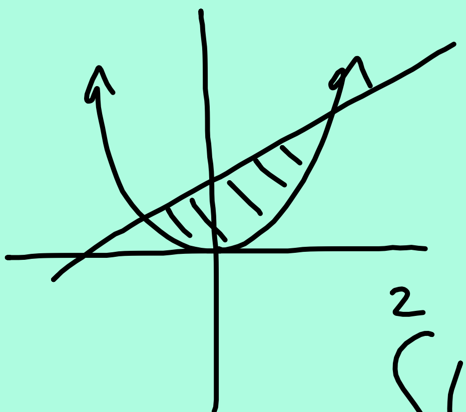
3) $\frac{1}{2\pi}e^{\pi x^2}+C$

4) $\frac{3\sqrt{2}}{2}\arctan(t\sqrt{2})+C$

$$\frac{3}{2}\int \frac{1}{t^2+\frac{1}{2}}\,dt = \frac{3}{2}\cdot\sqrt{2}\arctan\frac{t}{\sqrt{\frac{1}{2}}}+C$$

(27) $f(y) = y^2$

$g(y) = y + 2$



$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, -1$

$$\int_{-1}^{2} (y + 2 - y^2)\, dy$$

$$= \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_{-1}^{2}$$

$$= 2 + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 6 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3}$$

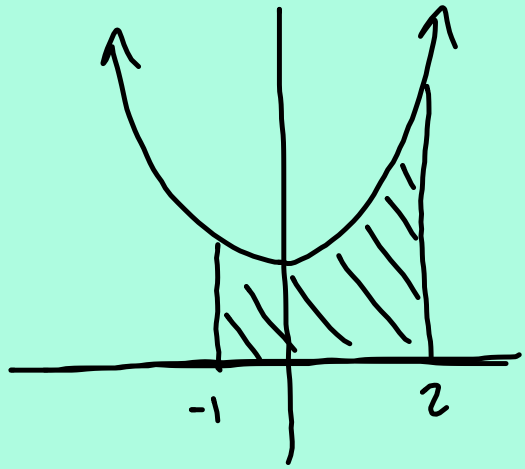
$$= 8 - 3 - \frac{1}{2} = \boxed{\frac{9}{2}}$$

(29) $f(y) = y^2 + 1$

$g(y) = 0$

$y = -1$

$y = 2$

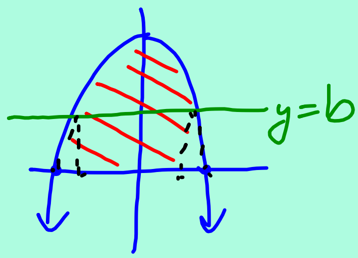


$$\int_{-1}^{2} (y^2 + 1)\,dy = \frac{1}{3}y^3 + y \Big|_{-1}^{2}$$

$$= \frac{8}{3} + 2 - \left(-\frac{1}{3} - 1\right)$$

$$= 3 + 2 + 1 = 6$$

(75) $y = 9 - x^2$, $y = 0$



$$\int_{-3}^{3} (9 - x^2)\,dx = 9x - \tfrac{1}{3}x^3 \Big|_{-3}^{3}$$

$$= 27 - 9 - (-27 + 9)$$

$$= 54 - 18 = 36$$

$x^2 = 9 - y$

$x = \pm\sqrt{9 - y}$

$$\int_0^b \left(\sqrt{9-y} - -\sqrt{9-y}\right) dy = 18$$

$$= \int_0^b 2\sqrt{9-y}\,dy = 18$$

$$= -2 \cdot \tfrac{2}{3}(9-y)^{3/2} \Big|_0^b = 18$$

$$= -\tfrac{4}{3}(9-b)^{3/2} - -\tfrac{4}{3}(27) = 18$$

$$-\tfrac{4}{3}(9-b)^{3/2} + 36 = 18$$

$$-\tfrac{4}{3}(9-b)^{3/2} = -18$$

$$(9-b)^{3/2} = \tfrac{27}{2}$$

$$9 - b = \left(\tfrac{27}{2}\right)^{2/3}$$

$$b = 9 - \left(\tfrac{27}{2}\right)^{2/3}$$

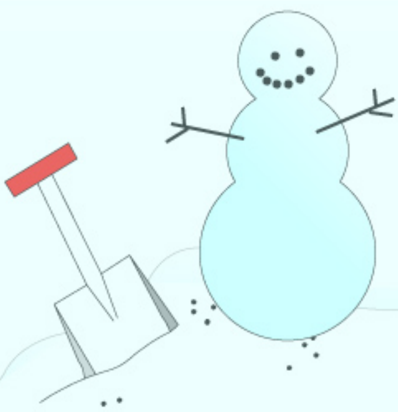
$$\approx 3.330$$

7.2a Volumes by Disks and Washers!!

At the end of this lesson you will be able to:

- Calculate the volume of a solid using the disk method
- Calculate the volume of a solid using the washer method

Revolve a function about an axis?!? What does this look like?



Can you derive the formula for the volume of a solid? Remember that with area, we broke curves up into rectangles. If we are revolving about an axis to create the solid, what shape could we use to estimate its volume?

Disk method:

$$V = \pi \int_a^b radius^2 \,(dx \text{ or } dy)$$

✓

if revolving about a horizontal axis => dx

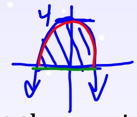
if revolving about a vertical axis => dy

radius = top - bottom (if dx)

radius = right - left (if dy)

★

Ex) Find the volume of the solid formed by rotating the graph bounded by $y = 4 - x^2$ and:

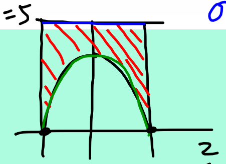


a) y = 0 about the x-axis
b) y = 5, x = -2 and x = 2 about y = 5
c) y = 0 and x = 0 (1st quadrant) about the y-axis
d) y = 0, y = 4 and x = 4 (1st quadrant) about x = 4

$4 - x^2 = 0$
$x = \pm 2$

a) $V = \pi \int_{-2}^{2} (4 - x^2 - 0)^2 \, dx = \frac{512}{15}\pi$ or 34.133π or 107.233

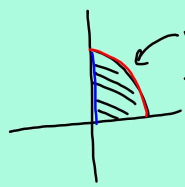
b)



$$V = \pi \int_{-2}^{2} \left(5 - (4 - x^2)\right)^2 dx$$

$= \frac{412}{15}\pi \approx 27.466\pi \approx 27.467\pi$
≈ 86.289

c)

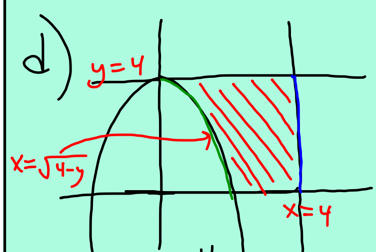


$y = 4 - x^2$
$x^2 = 4 - y$
$x = \pm\sqrt{4-y}$

$$V = \pi \int_{0}^{4} \left(\sqrt{4-y} - 0\right)^2 dy$$

$= 8\pi \approx 25.132$ or 25.133

d)



$$V = \pi \int_{0}^{4} \left(4 - \sqrt{4-y}\right)^2 dy$$

You try! (Calculators permitted: just show setup and answer)

Find the volume of the solid formed by revolving the region bounded by $y = 2x^3$ and:

a) $y = 0$ and $x = 2$ about the x-axis

b) $x = 2$ and $y = -2$ about $y = -2$

c) $x = 0$ and $y = 18$ about the y-axis

d) $y = 0$, $y = 18$ and $x = 6$ about $x = 6$

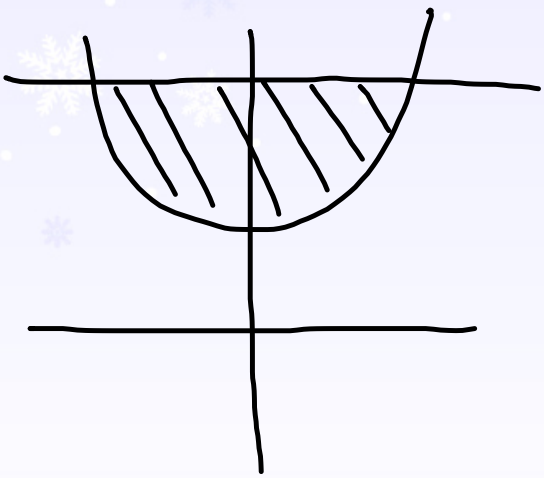
$$a)\ \pi\int_0^2 (2x^3)^2\,dx = \frac{512}{7}\pi \approx 229.785$$

$$b)\ \pi\int_{-1}^2 (2x^3 - -2)^2\,dx = \frac{810}{7}\pi \approx 363.527$$

$$c)\ \pi\int_0^{18} \left(\sqrt[3]{\frac{1}{2}y}\right)^2 dy = \frac{162\pi\sqrt[3]{3}}{5} \approx 146.803$$

$$d)\ \pi\int_0^{18} \left(6 - \sqrt[3]{\frac{1}{2}y}\right)^2 dy \approx 1123.921$$

Grow your brain! In your groups, create an integral that will give you the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ about the x-axis.



Washer method!

$$V = \pi \int_a^b (R^2 - r^2)(dx \; or \; dy)$$

horizontal axis of rotation => dx

vertical axis of rotation => dy

R = 'outer function' and axis of revolution

r = 'inner function' and axis of revolution

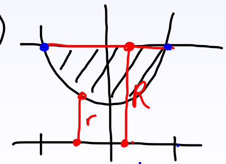
dx: top - bottom

dy: right - left

Ex) Find the volume of the solid formed by rotating the graph bounded by $y = x^2 + 1$ and:

a) $y = 2$ about the x-axis
b) $y = 2$ about $y = 5$
c) $y = 2$ about the $y = -3$
d) $x = 0$ and $y = 2$ (1st quadrant) about $x = 4$

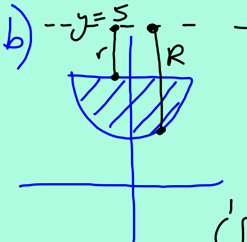
$x^2+1=2 \quad x^2=1 \quad x=\pm 1$

a)


$R = 2-0 = 2$ (outer axis)

$r = x^2+1-0 = x^2+1$ (inner axis)

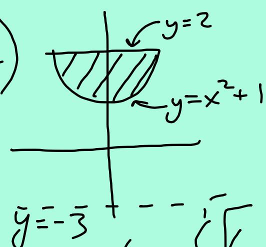
$$V = \pi \int_{-1}^{1} \left[(2)^2 - (x^2+1)^2 \right] dx$$

b)


$R = 5-(x^2+1) = 4-x^2$

$r = 5-2 = 3$

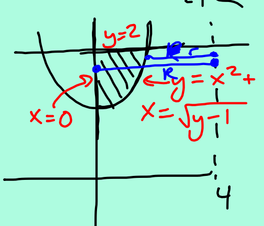
$$V = \pi \int_{-1}^{1} \left[(4-x^2)^2 - (3)^2 \right] dx$$

c)


$R = 2 - -3 = 5$

$r = x^2+1 - -3 = x^2+4$

$$V = \pi \int_{-1}^{1} \left[(5)^2 - (x^2+4)^2 \right] dx$$

d)


$R = 4-0 = 4$

$r = 4-\sqrt{y-1}$

$$V = \pi \int_{1}^{2} \left[(4)^2 - (4-\sqrt{y-1})^2 \right] dy$$

You try! (Calculators permitted: just show setup and answer)

Find the volume of the solid formed by revolving the region bounded by $y = 4x + x^2$ and $y = x$ about:

a) $y = 0$

b) $y = -5$

$$a)\ \pi \int_{-3}^{0} \left(\left(0 - (4x + x^2)\right)^2 - (0 - x)^2 \right) dx = \frac{108}{5}\pi \approx 67.858$$

✓

$$b)\ \pi \int_{-3}^{0} \left((x - -5)^2 - \left((4x + x^2) - -5\right)^2 \right) dx = \frac{117}{5}\pi \approx 73.513$$

What have we learned?

- Can I find the volume formed by revolving a region about an axis using disk method?
- Can I find the volume formed by revolving a region about an axis using washer method?