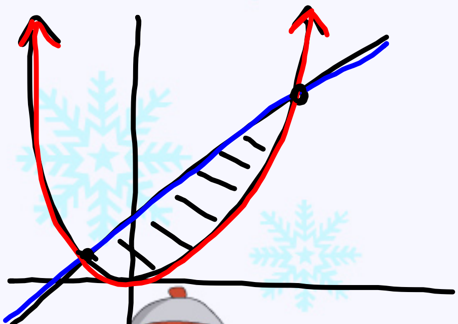
Let's warm up with a warmup!!

Find the area of the region bounded by $y = x^2$ and $y = 2x + 3$. (You may use a calculator.)



$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

$$\int_{-1}^{3} (2x + 3 - x^2)\,dx$$

✓ area = 32/3 or 10.667

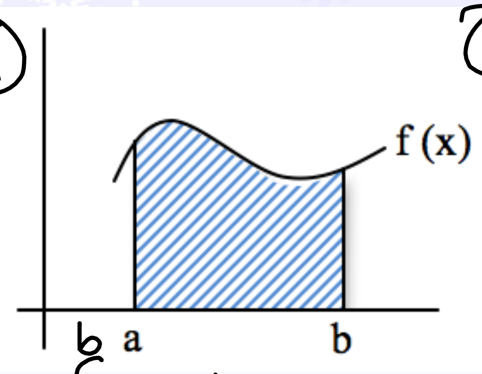
7.1 Area!!

At the end of this lesson you will be able to:

- Calculate the area between curves

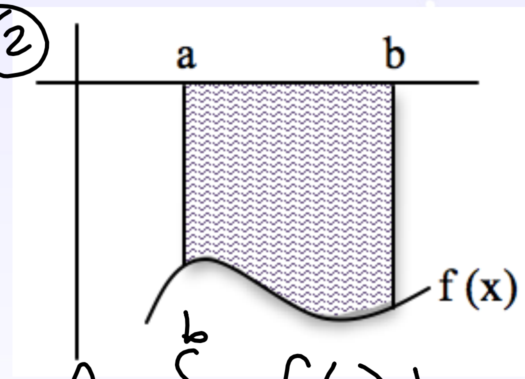
Warmup #2: Write an integral that will result in the area of the shaded region

(1)



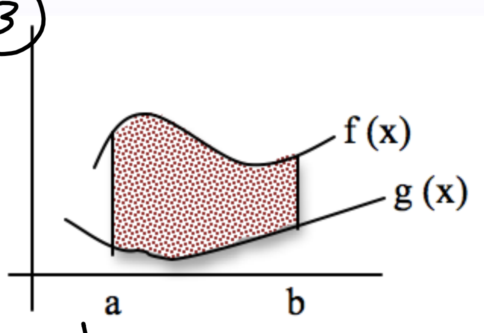
$$A = \int_a^b f(x)\,dx$$

(2)



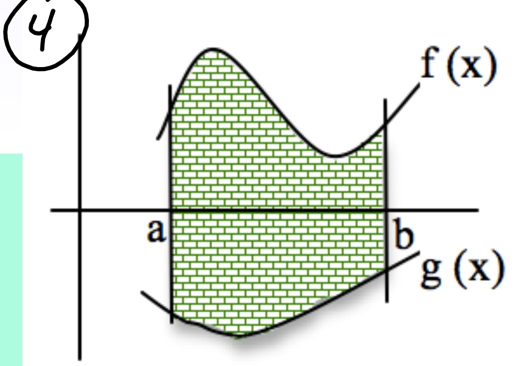
$$A = \int_a^b -f(x)\,dx$$

(3)



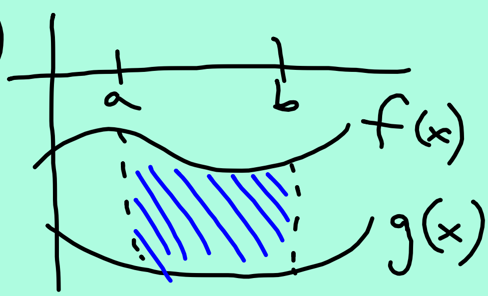
$$A = \int_a^b [f(x) - g(x)]\,dx$$

(4)



$$A = \int_a^b [f(x) - g(x)]\,dx$$

(5)



$$A = \int_a^b [f(x) - g(x)]\,dx$$

In general, the area between 2 curves is given by:

$$\text{Area} = \int_a^b [\text{top curve} - \text{bottom curve}]\, dx$$

With no exceptions! (Unless we decide to turn the curves sideways, then it's right - left, but apart from that, no exceptions!) Of course this means that if the curves intersect you will need to split up the integral to maintain a 'top - bottom' status at all times. :)

You try! Find the area of the total region(s) bounded by $y = x^3 + 3x^2$ and $y = 4x^2 + 6x$ (calculators may be used for arithmetic only)

$$x^3+3x^2=4x^2+6x$$

$$x^3-x^2-6x=0$$

$$x(x-3)(x+2)=0$$

$$x=0,3,-2$$

$$\text{Area}=\int_{-2}^{0}\left[x^3+3x^2-(4x^2+6x)\right]dx$$

$$+\int_{0}^{3}\left[4x^2+6x-(x^3+3x^2)\right]dx$$

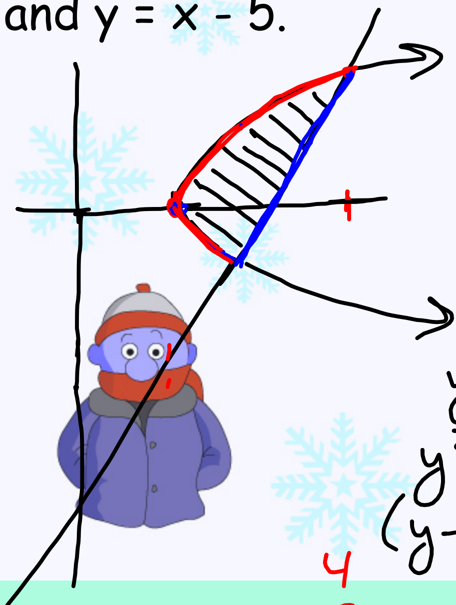
$$=\int_{-2}^{0}(x^3-x^2-6x)\,dx+\int_{0}^{3}(x^2+6x-x^3)\,dx$$

$$=\frac{1}{4}x^4-\frac{1}{3}x^3-3x^2\Big|_{-2}^{0}+\frac{1}{3}x^3+3x^2-\frac{1}{4}x^4\Big|_{0}^{3}$$

$$=0-\left(4+\frac{8}{3}-12\right)+9+27-\frac{81}{4}-0$$

$$=8-\frac{8}{3}+36-\frac{81}{4}=\frac{253}{12}\approx 21.083$$

Let's try one together! Find the area of the region bounded by $y = \sqrt{2x - 2}$, $y = -\sqrt{2x - 2}$ and $y = x - 5$.



$$x = \frac{1}{2}(y^2+2)$$

$$x = y + 5$$

$$\frac{1}{2}(y^2+2) = y + 5$$

$$y^2 + 2 = 2y + 10$$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0 \qquad y = 4, -2$$

$$A = \int_{-2}^{4} \left[y + 5 - \frac{1}{2}(y^2+2) \right] dy$$

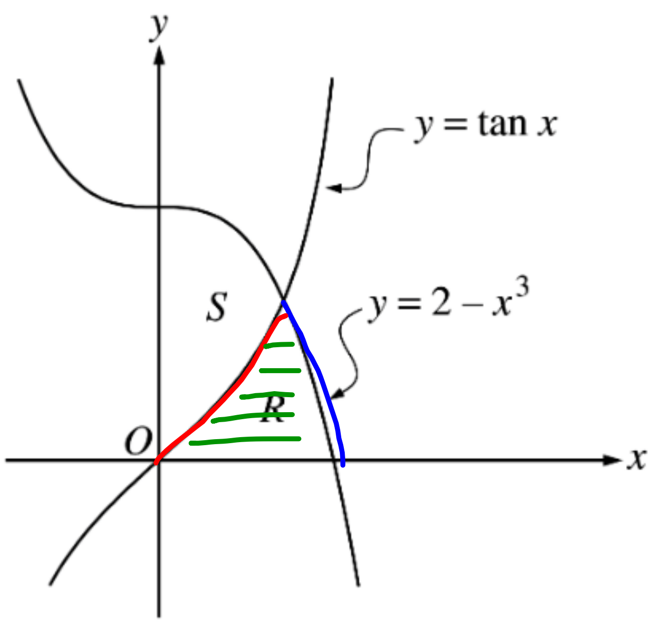
$$A = \int_{-2}^{4} \left(y + 4 - \frac{1}{2}y^2 \right) dy$$

$$= \frac{1}{2}y^2 + 4y - \frac{1}{6}y^3 \Big|_{-2}^{4}$$

$$= 8 + 16 - \frac{32}{3} - \left(2 - 8 + \frac{4}{3} \right)$$

$$= 24 - \frac{32}{3} + 6 - \frac{4}{3}$$

$$= 30 - \frac{36}{3} = \boxed{18}$$



2001 #1 (calc permitted)

Let R and S be the regions in the first quadrant shown in the figure. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

a) Find the area of R

b) Find the area of S

Let's do some review! (calculators permitted)

A particle moves along the x-axis in such a way that its velocity is $v(t) = -e^{0.5t} + 2t$ on the interval $0 \le t \le 4$. When t = 0, the position of the particle is x = -2.

a) When does the particle change direction on $0 \le t \le 4$? Why?

b) What is the total distance traveled by the particle on $0 \le t \le 4$?

c) What is the position of the particle at t = 1?

d) What is the particle's position when it is farthest to the left on $0 \le t \le 4$?

a) The particle changes direction at $t \approx 0.715$ b/c v(t) changes sign

✓

b) Total distance = $\int_0^4 |v(t)|\,dt \approx 3.918$

c) $\int_0^1 v(t)dt = x(1) - x(0)$

so

$$x(1) = \int_0^1 v(t)dt + x(0) \approx -0.29744 + -2 \approx -2.297$$

d) The particle is farthest left at $t \approx 0.715$ so we need x(0.715)

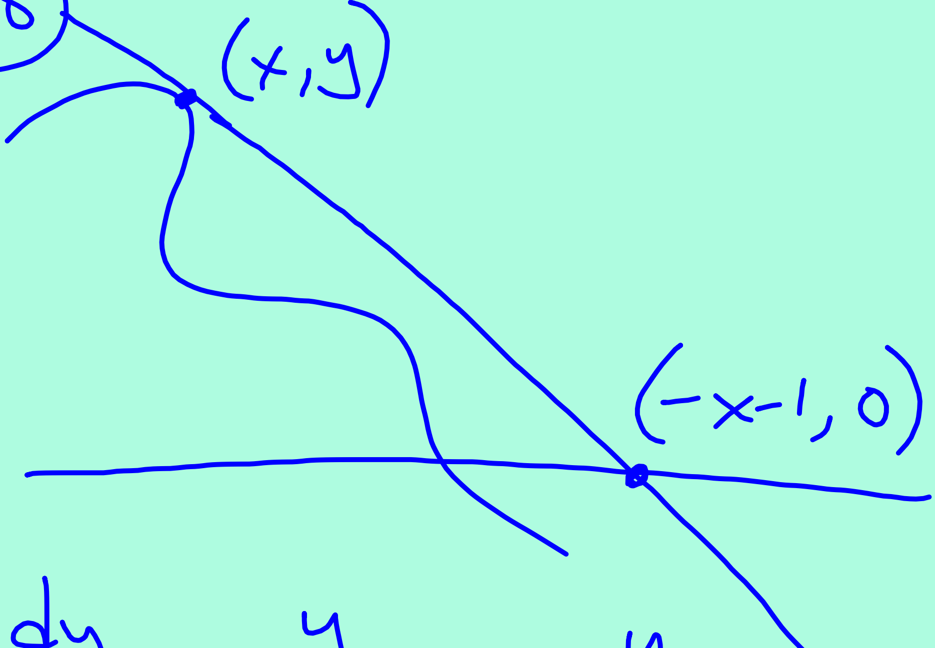
$$\int_0^{0.71480591} v(t)dt = x(0.715) - x(0)$$

$$x(0.715) = \int_0^{0.71480591} v(t)dt + x(0) \approx -0.3482762 + -2 \approx -2.348$$

What have we learned?

- Can I find the area between 2 curves, even if they cross over each other somewhere in the interval?

(18)



$$\frac{dy}{dx} = \frac{y}{x-(-x-1)} = \frac{y}{2x+1}$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{2x+1}\,dx$$

$$\ln|y| = \frac{1}{2}\ln|2x+1| + C$$

January 04, 2016