WARMUP!!

Suppose A = Pe^{rt}.

If A = 100 when t = 0, and A = 500 when t = 2, solve for P and r and rewrite the function. Calculators are NOT permitted.

$A = Pe^{r \cdot t}$

$100 = Pe^{0} = P$

$A = 100e^{r \cdot t}$

$500 = 100e^{2r}$

$5 = e^{2r}$

$\ln 5 = 2r$

$r = \frac{1}{2}\ln 5$

$A = 100e^{\left(\frac{1}{2}\ln 5\right)\cdot t}$

19 $\displaystyle\int \frac{x+5}{\sqrt{9-(x-3)^2}}\,dx$

$u = 9-(x-3)^2$

$du = -2(x-3)\,dx \longrightarrow -\frac{1}{2}du = (x-3)\,dx$

$= \displaystyle\int \frac{x-3}{\sqrt{9-(x-3)^2}}\,dx + \int \frac{8}{\sqrt{9-(x-3)^2}}\,dx$

$= \displaystyle\int -\frac{1}{2}u^{-\frac{1}{2}}\,du + 8\arcsin\frac{x-3}{3} + C$

$= -u^{\frac{1}{2}} + 8\arcsin\frac{x-3}{3} + C$

$= -\sqrt{9-(x-3)^2} + 8\arcsin\frac{x-3}{3} + C$

(43) $\displaystyle\int \sqrt{e^t - 3}\,dt$

$u = \sqrt{e^t - 3} \longrightarrow u^2 = e^t - 3$

$e^t = u^2 + 3$

$du = \frac{1}{2}(e^t - 3)^{-\frac{1}{2}}(e^t)\,dt$

$du = \dfrac{e^t}{2\sqrt{e^t - 3}}\,dt \qquad \dfrac{2\sqrt{e^t-3}}{e^t}\,du = dt$

$= \displaystyle\int u \cdot \frac{2u}{u^2+3}\,du \qquad \frac{2u}{u^2+3}\,du = dt$

$= \displaystyle\int \frac{2u^2}{u^2+3}\,du$

$$\begin{array}{r|l} & \quad 2 \\ \hline u^2+3 & 2u^2 + 0 \\ & 2u^2 + 6 \\ \hline & \quad -6 \end{array}$$

$= \displaystyle\int 2 - \frac{6}{u^2+3}\,du$

$= 2u - \dfrac{6}{\sqrt{3}}\arctan\dfrac{u}{\sqrt{3}} + C$

$= 2\sqrt{e^t - 3} - \dfrac{6}{\sqrt{3}}\arctan\dfrac{\sqrt{e^t - 3}}{\sqrt{3}} + C$

6.2 Exponential Growth and Decay!

Essential Learning Targets:

> Writes the model for exponential growth and decay arising from the statement "The rate of change of a quantity is proportional to the size of the quantity" as dy/dt = ky

> Uses antidifferentiation to find specific solutions to differential equations with given initial conditions, including applications to exponential growth and decay

What concept do we need to know going in?

Proportionality!

Y is directly proportional to x: $y = kx$

Y is inversely proportional to x: $y = k/x$

ex) The rate of change $\frac{dP}{dt}$ of the population, P, of Lazytown is directly proportional to the population at any given time, t.

$=$ $k\cdot$ P

Rewrite this statement using calculus notation

$$\frac{dP}{dt} = k\cdot P$$

Separate the variables and solve the differential equation

$$\int \frac{1}{P}\,dP = \int k\,dt$$

$$\ln P = kt + C$$

$$P = e^{kt+C} = e^{kt}\cdot e^{C}$$

$$\boxed{P = Me^{kt}}$$

What does this solution remind you of?

$$A = Pe^{rt} \qquad y = Ce^{kt}$$

Now for the precalc part: If the population of Lazytown in 1930 was 50,000 and in 1960 it increased to 75,000, what is the expected population in 2020? Calculators permitted.

t	P
0	50000
30	75000
90	?

let t=0 be 1930

$$50000 = Me^0 = M$$

$$P = 50000e^{kt}$$

$$75000 = 50000e^{30k}$$

$$\frac{3}{2} = e^{30k}$$

$$\ln\left(\frac{3}{2}\right) = 30k$$

$$k = \frac{1}{30}\ln\left(\frac{3}{2}\right)$$

Keep k exact

$$P = 50000e^{\left[\frac{1}{30}\ln\left(\frac{3}{2}\right)\right]\cdot t}$$

$$P(90) = 50000e^{\frac{1}{30}\ln\left(\frac{3}{2}\right)\cdot 90}$$

$$= \underline{168750}$$

You try! The rate of decay of radium is proportional to the amount present at any time.

Rewrite this using calculus notation, separate the variables, and solve the differential equation.

$$\frac{dR}{dt} = KR$$

$$\int \frac{1}{R}\,dR = \int K\,dt$$

$$\ln R = Kt + C$$

$$R = e^{Kt+C}$$

$$R = Be^{Kt}$$

If 60 mg of radium are present now and its half-life is 1690 years, how much radium will be present 100 years from now? (Calculators permitted)

t	R
0	60
1690	30
100	?

$$60 = Be^0 = B$$

$$R = 60e^{Kt}$$

$$30 = 60e^{1690K}$$

$$K = \frac{1}{1690}\ln\left(\frac{1}{2}\right)$$

$$R = 60e^{\frac{1}{1690}\ln\left(\frac{1}{2}\right)t}$$

$$R(100) = 60e^{\frac{100}{1690}\ln\left(\frac{1}{2}\right)}$$

$$\approx 57.5889 \text{ mg}$$

Can you figure this one out?

In a certain culture where the rate of growth of bacteria is proportional to the amount present, the number triples in 5 hours. If at the end of 12 hours there were 10 million bacteria, how many were present initially? (Calculators permitted)

$$B = Ce^{kt}$$

$$3\cancel{C} = \cancel{C}e^{5k}$$

$$\ln 3 = 5k$$

$$k = \frac{\ln 3}{5}$$

$$B = Ce^{\frac{1}{5}\ln 3 \cdot t}$$

$$10000000 = Ce^{\frac{12}{5}\ln 3}$$

$$C \approx 715993.3499$$

t	B
12	10000000
0	C
5	3C

Newton's Law of Cooling!!

The rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium.

$\frac{dT}{dt} = K \cdot (T-M)$

T = temp of object

t = time

M = temp of medium (constant)

$$\frac{dT}{dt} = K(T-M)$$

ex) Mrs. Price bakes a pie at 350°F. When the pie is baked to golden perfection, she takes it out of the oven and sets it on the counter. The temperature of the kitchen is 70°F. After 10 minutes, she checks the temperature and the pie has cooled to 250°F. How long will it take for the pie to cool to 80°F?

$$\frac{dT}{dt} = K(T-70)$$

$$\int \frac{1}{T-70}\,dT = \int K\,dt$$

$$\ln(T-70) = Kt + C$$

$$T-70 = e^{Kt+C} = Qe^{Kt}$$

$$T = Qe^{Kt} + 70$$

$$350 = Qe^{0} + 70$$

$$Q = 280$$

$$T = 280e^{Kt} + 70$$

$$250 = 280e^{10K} + 70$$

$$180 = 280e^{10K}$$

$$\frac{9}{14} = e^{10K}$$

$$K = \frac{1}{10}\ln\left(\frac{9}{14}\right)$$

$$T = 280e^{\frac{1}{10}\ln\left(\frac{9}{14}\right)t} + 70$$

$$80 = 280e^{\frac{1}{10}\ln\left(\frac{9}{14}\right)t} + 70$$

$$\frac{1}{28} = e^{\frac{1}{10}\ln\left(\frac{9}{14}\right)t}$$

$$\ln\frac{1}{28} = \frac{1}{10}\ln\frac{9}{14}\cdot t$$

$$t = \frac{\ln\frac{1}{28}}{\frac{1}{10}\ln\frac{9}{14}} \approx 75.4177 \text{ minutes}$$

If there is time, try this one: If an amount of money invested doubles itself in 10 years at interest compounded continuously, what is the rate of interest? How long will it take for the original amount to triple itself?

What may I assume for homework?

For all radioactive decay and continuously compounded interest problems, you may safely assume that $y = Ce^{kt}$. You do not have to re-derive it with every problem. However, as you get further into the homework, be aware that not every situation will follow this model.

What have we learned?

- Can I write a differential equation based on given information?
- Can I solve problems related to exponential growth and decay?