Let's warm up with a warmup!!

1) $\int \frac{4}{x^2+9}\,dx = \frac{4}{3}\arctan\frac{x}{3} + C$ ✓

$x^2 + a^2$

2) $\int \frac{4x}{x^2+9}\,dx = 2\ln(x^2+9) + C$

3) $\int \frac{4x^2}{x^2+9}\,dx = \int\left(4 - \frac{36}{x^2+9}\right)dx = 4x - 12\arctan\frac{x}{3} + C$

4) $\int \frac{4x^3}{x^2+9}\,dx \quad \int\left(4x - \frac{36x}{x^2+9}\right)dx = 2x^2 - 18\ln(x^2+9) + C$

6.1b Euler's Method!!

At the end of this lesson you will be able to:

- Use Euler's Method to create a list of points that approximate the solution to a differential equation

How does Euler's Method work? Let's see!

Let's walk through an example slowly:

ex) Suppose $y' = 2x + y$. If $y(0) = 1$, approximate $y(1)$ using Euler's Method with $\Delta x = 0.25$.

$x = 0, .25, .5, .75, 1$

a) Write the equation of the first tangent line:

point: (0, 1) slope: $2(0) + 1 = 1$

$y - 1 = 1(x - 0)$, so $y = x + 1$

b) Use this to find the next point: (0.25, 1.25)

c) Find the equation of the second tangent line:

point: (0.25, 1.25) slope: $2(0.25) + 1.25 = 1.75$

$y - 1.25 = 1.75(x - 0.25)$

d) Use this to find the next point: (0.5, 1.6875)

e) point: (0.5, 1.6875)

slope: $2(.5) + 1.6875 = 2.6875$

$y - 1.6875 = 2.6875(x - 0.5)$

f) point(0.75, 2.359375) slope: 3.859375

$y - 2.359375 = 3.859375(x - 0.75)$

final point: (1, 3.32421875)

so $y(1) \approx 3.324$

There is another way to do the same thing:
you can memorize the following and make a table (you can see how this is derived from the previous slide). (BTW, this is the method used by the College Board.)

$$y_{n+1} = y_n + (\Delta x)(y'(x_n, y_n))$$

Let's try the last example again:

Suppose $y' = 2x + y$, $y(0) = 1$ and $\Delta x = 0.25$.
Use Euler's method to approximate $y(1)$.

n	x	y
0	0	1
1	0.25	1+.25(2(0) + 1) =1.25
2	0.5	1.25 + .25(2(.25) +1.25) = 1.6875
3	0.75	1.6875 + .25(2(.5) + 1.6875) = 2.359375
4	1	2.359375 + .25(2(.75) + 2.359375) = 3.32421875

so $y(1) \approx 3.324$

You try! Use Euler's method to approximate the solution to y' = ln(xy) if y(1) = 2 and h = 0.2 using 5 steps.

$$y_{n+1} = y_n + \Delta x(y'(x_n, y_n))$$

n	x	y
0	1	2
1	1.2	2 + 0.2ln(2) ≈ 2.138629
2	1.4	2.138629+.2ln((1.2)(2.138629)) ≈ 2.327127
3	1.6	2.327127 + .2ln((1.4)(2.327127)) ≈ 2.563348
4	1.8	2.563348 + .2ln((1.6)(2.563348)) ≈ 2.854612
5	2	2.854612 + .2ln((1.8)(2.854612)) ≈ 3.172325

so y(2) ≈ 3.172

You try again! BC 2009 #4 parts a and c

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2 y$

Let y = f(x) be a particular solution to the differential equation with the initial condition f (-1) = 2.

(a) Use Euler's method with two steps of equal size, starting at x = -1, to approximate f(0). Show the work that leads to your answer.

(c) Find the particular solution y = f(x) to the given differential equation with the initial condition f(-1) = 2.

One last one to get you ready for your homework: Complete the table using the exact solution of the differential equation and two approximations obtained using Euler's method to approximate the particular solution of the differential equation. Use h = 0.2 and 0.1 and compute each approximation to four decimal places.

$dy/dx = 2x/y$

initial condition: (0, 2)

exact solution: $y = \sqrt{2x^2 + 4}$

x	0	0.2	0.4	0.6	0.8	1
y (exact)	2	2.0199	2.0785	2.1726	2.2978	2.4495
y (h=0.2)						
y (h=0.1)						

What have we learned?

- What is Euler's Method used for?
- What are the two different ways to apply Euler's Method?
- Can I successfully use one (or both) of these methods regardless of how the problem is worded?

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