Let's warm up with a warmup!!

Find the general solution to the differential equation, $3x + 2yy' = 0$. Then find the particular solution, $\boxed{y = f(x)}$, that passes through (1, -3).

$$3x + 2y\frac{dy}{dx} = 0$$

$$\int 2y\,dy = \int -3x\,dx$$

$$y^2 = \frac{-3}{2}x^2 + C$$

$$9 = \frac{-3}{2} + C$$

$$C = \frac{21}{2}$$

$$y^2 = -\frac{3}{2}x^2 + \frac{21}{2}$$

not a function $\rightarrow$ $y = \pm\sqrt{-\frac{3}{2}x^2 + \frac{21}{2}}$

$$y = -\sqrt{-\frac{3}{2}x^2 + \frac{21}{2}}$$

based on (1, -3)

general solution: $y^2 = -3/2\,x^2 + C$

particular solution: $y = -\sqrt{-3/2\,x^2 + 21/2}$

6.1 Slope Fields!!

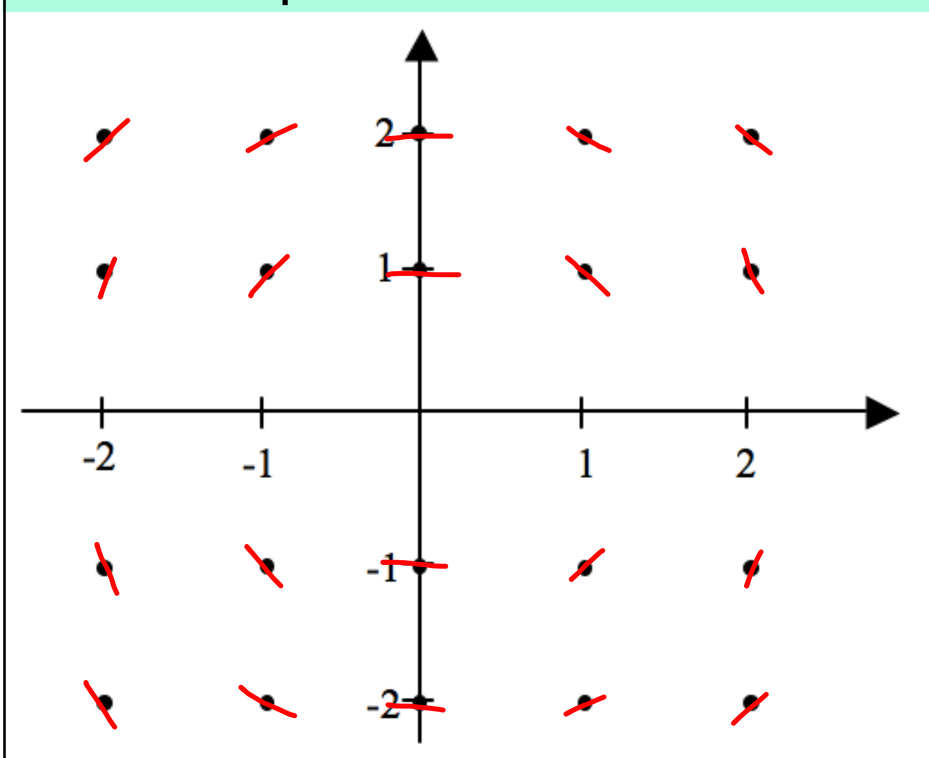
At the end of this lesson you will be able to:

- Sketch a slope field when given a differential equation
- Sketch the solution to a differential equation on a slope field
- Match a slope field with its corresponding differential equation

What is a slope field?

A slope field is a graph consisting of line segments, with each segment drawn using the slope of the function at that point. When a large number of them are put together, they give a visual representation of a large number of solutions to a differential equation at one time.

ex) Sketch a slope field for $\frac{dy}{dx} = -\frac{x}{y}$ at the indicated points below.



x, y	$\frac{dy}{dx}$
$(1,1)$	-1
$(1,2)$	$-\frac{1}{2}$
$(2,1)$	-2
$(2,2)$	-1

For the next several problems, do the following:
1. Determine the differential equation being graphed by each of the slope fields.
2. Sketch two solutions, one of which passes through the indicated point.

Note: **The sketch of a solution through a point must meet the following criteria:**
1. It must pass through the given point
2. It must follow the general trend of the field
3. It must extend as far as possible within its domain, or to the edge of the given window

ex)

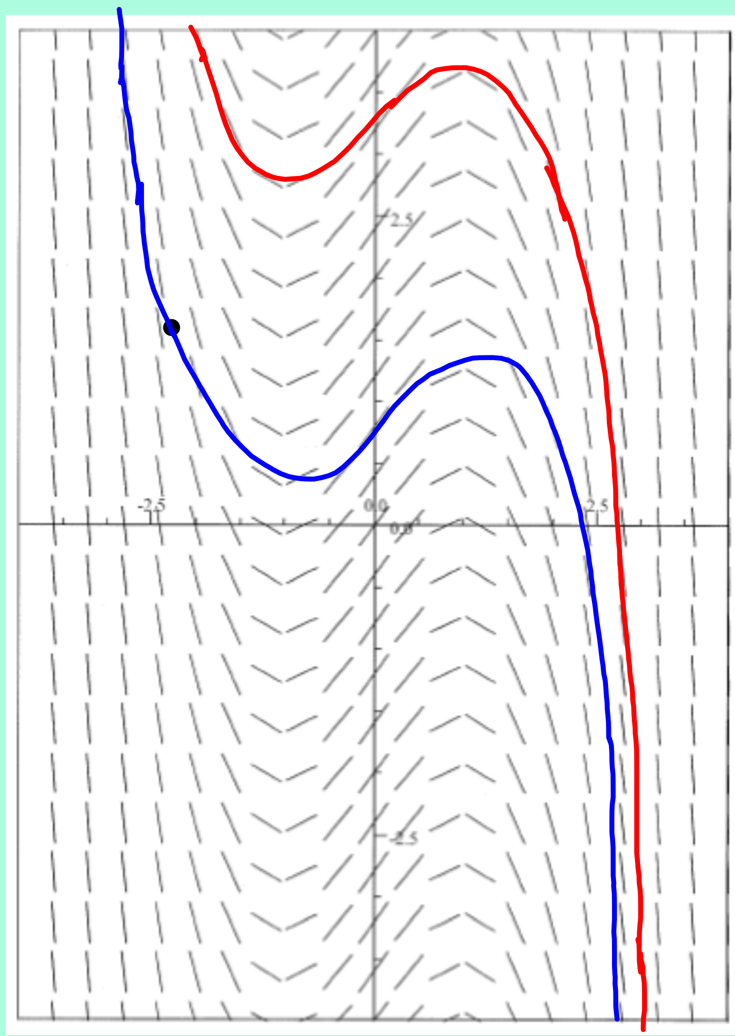
~~a)~~ $\dfrac{dy}{dx} = 1 - x^3$

b) $\dfrac{dy}{dx} = 1 - x^2$

~~c)~~ $\dfrac{dy}{dx} = x + y$

~~d)~~ $\dfrac{dy}{dx} = \dfrac{x}{y}$

~~e)~~ $\dfrac{dy}{dx} = \ln x$



You try!

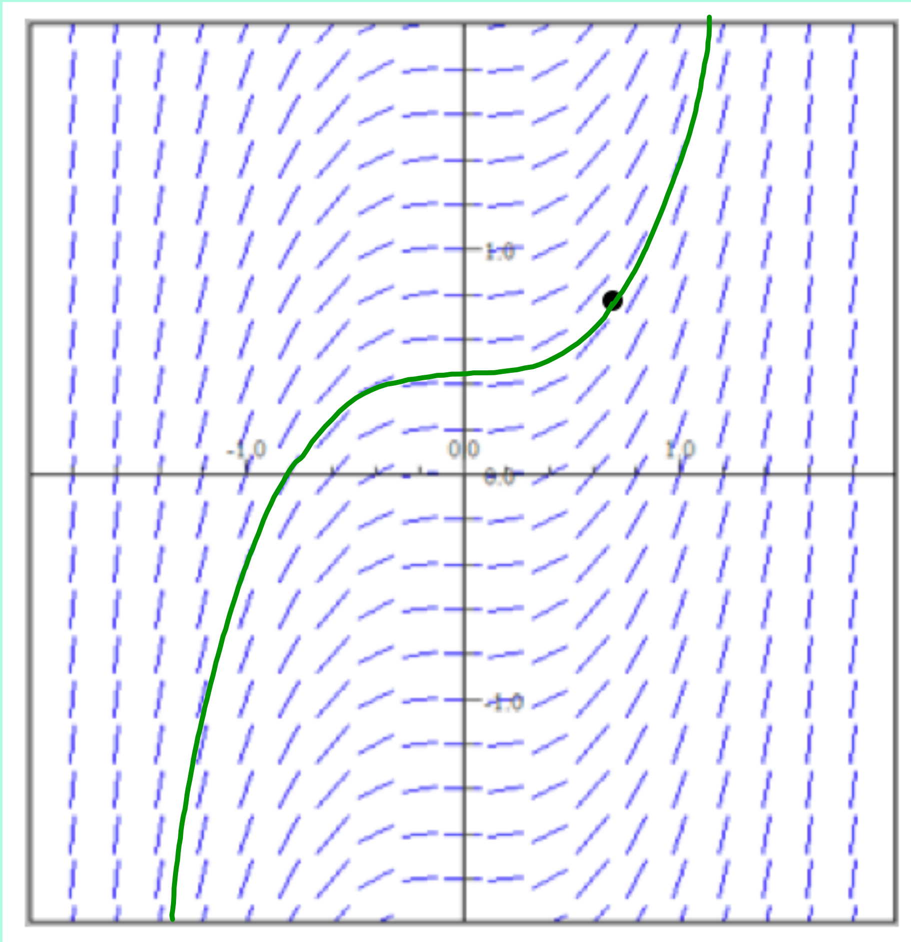
a) $\frac{dy}{dx} = x^3$

b) $\frac{dy}{dx} = 3x^2$

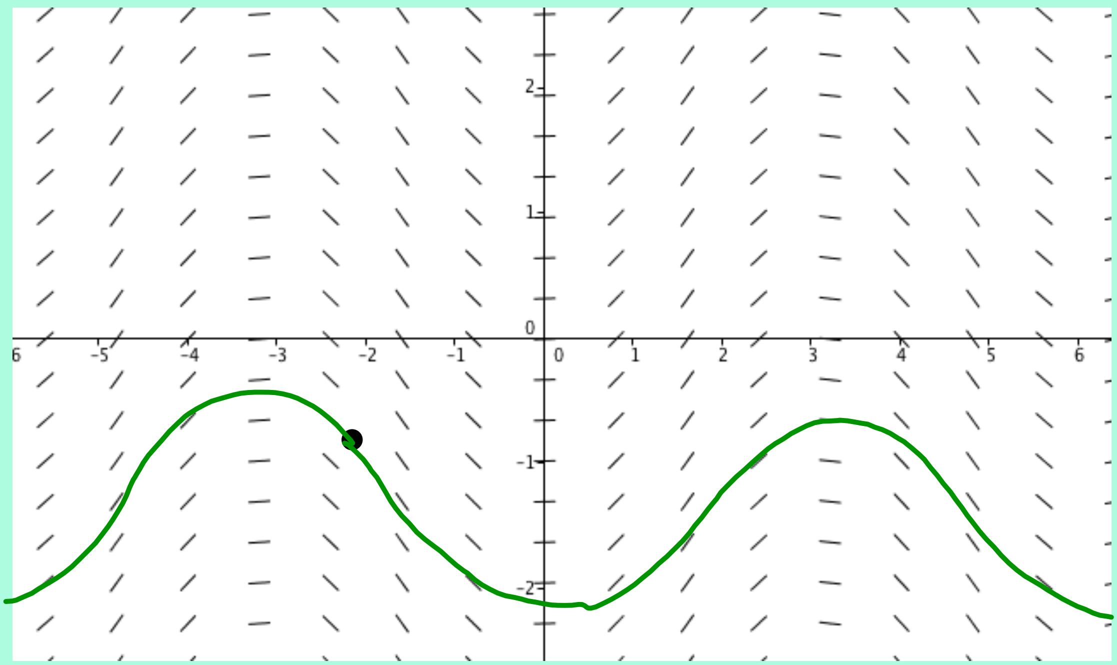
c) $\frac{dy}{dx} = 2x + y$

d) $\frac{dy}{dx} = \frac{x}{y}$

e) $\frac{dy}{dx} = \ln x$



You try again!



a) $dy/dx = x^3$ b) $dy/dx = x^2$ c) $dy/dx = \sin x$

d) $dy/dx = \cos x$ e) $dy/dx = \tan x$

The **domain of the particular solution** to a differential equation must meet the following requirements:

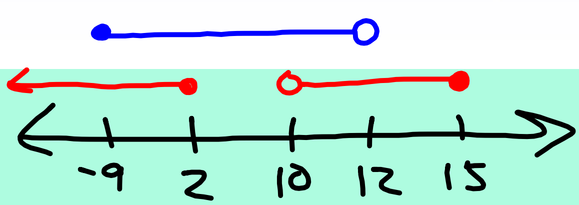
a) It must satisfy the domain of the particular solution

b) It must satisfy the domain of the original differential equation

c) It must be the largest <u>single</u> interval that contains the given point

Think about it:

Suppose the domain of dy/dx is $(-\infty, 2] \cup (10, 15]$ and the domain of $y = f(x)$ is $[-9, 12)$. What would the domain of the solution be that contains a point at $\boxed{x = 11?}$

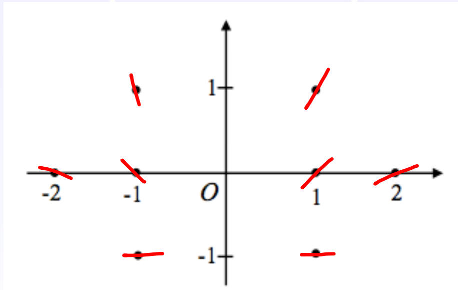


$\cancel{[-9, 2]} \cup \boxed{(10, 12)}$

Consider the differential equation dy/dx = (1 + y)/x, where x ≠ 0.

a) Sketch a slope field for the differential equation at the points indicated below.

$\frac{dy}{dx} = \frac{1+y}{x}$



b) Find the equation of the line tangent to f(x) at the point (-1, 1) and use it to approximate f(-1.1).

c) Find the particular solution, y = f(x), to the given differential equation with the initial condition, f(-1) = 1.

d) State the domain of the solution found in part c.

b) $f(-1)=1$

$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{1+1}{-1} = -2$

$y-1 = -2(x+1)$

$y = -2(x+1)+1$

so $f(-1.1) \approx -2(-1.1+1)+1 = 1.2$

c) $\frac{dy}{dx} = \frac{1+y}{x}$

$\int \frac{1}{1+y}\,dy = \int \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + C$

$|1+y| = e^{\ln|x|+C} = Ae^{\ln|x|} = A|x|$

$1+y = M|x|$

$y = M|x| - 1$

$1 = M|-1| - 1$

$2 = M$

$\boxed{y = 2|x| - 1}$

d) domain of y: $(-\infty, \infty)$

domain of $\frac{dy}{dx}$: $(-\infty, 0) \cup (0, \infty)$

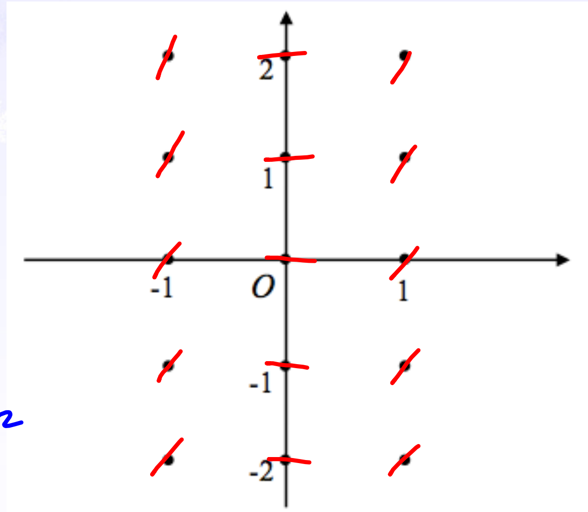
domain of solution: $\boxed{(-\infty, 0)}$

$(-1, 1)$

Consider the differential equation $dy/dx = 3x^2$.

a) Sketch a slope field for the differential equation at the points indicated below.

$\frac{dy}{dx} = 3x^2$



b) Find the equation of the line tangent to f(x) at the point (1, 5) and use it to approximate f(1.2).

c) Find the particular solution, y = f(x), to the given differential equation with the initial condition, f(1) = 5.

d) State the domain of the solution found in part c.

b) $\left.\frac{dy}{dx}\right|_{x=1} = 3$

$y - 5 = 3(x-1)$

$y = 3(x-1) + 5$

$f(1.2) \approx 3(1.2-1) + 5 = \boxed{5.6}$

c) $\int dy = \int 3x^2\,dx$

$y = x^3 + C$

$5 = 1 + C$

$C = 4$

$\boxed{y = x^3 + 4}$

d) domain of y: $(-\infty, \infty)$

domain of $\frac{dy}{dx}$: $(-\infty, \infty)$

domain of solution: $(-\infty, \infty)$

What have we learned?

- Can I sketch a slope field?
- Can I sketch the solution to a differential equation on a slope field?
- Can I match a slope field with its corresponding differential equation?