Let's warm up with a warmup!!

Find the general solution to the differential equation, $3x + 2yy' = 0$. Then find the particular solution, $\boxed{y = f(x)}$, that passes through $(1, -3)$.

$3x + 2y\frac{dy}{dx} = 0$

$\int 2y\,dy = \int -3x\,dx$

$y^2 = -\frac{3}{2}x^2 + C$

$9 = -\frac{3}{2} + C$

$C = \frac{21}{2}$

$y^2 = -\frac{3}{2}x^2 + \frac{21}{2}$

$y = - \sqrt{-\frac{3}{2}x^2 + \frac{21}{2}}$

general solution: $y^2 = -3/2\ x^2 + C$

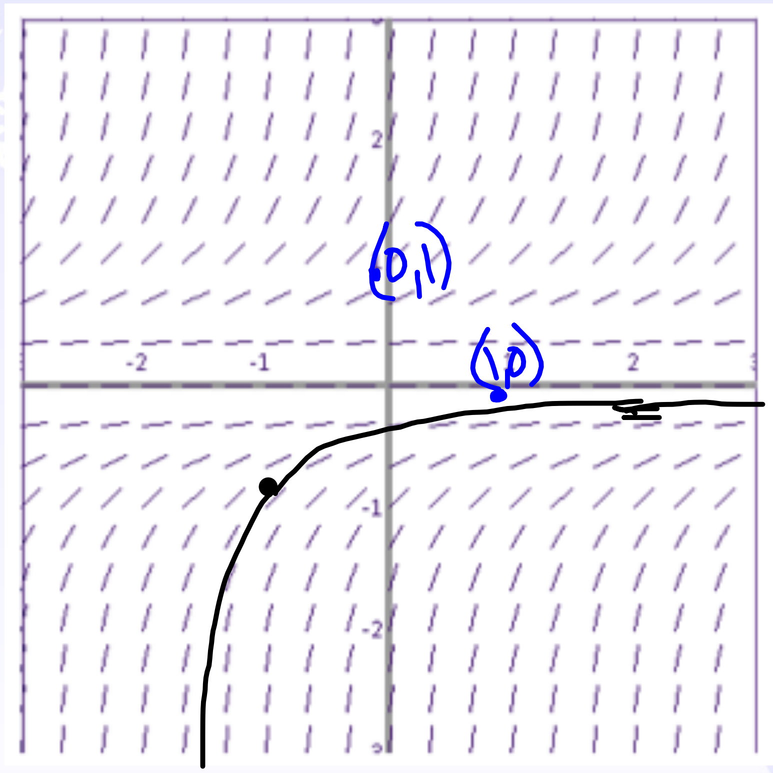
particular solution: $y = -\sqrt{-3/2\ x^2 + 21/2}$

6.1 Slope Fields!!

At the end of this lesson you will be able to:

- Sketch a slope field when given a differential equation
- Sketch the solution to a differential equation on a slope field
- Match a slope field with its corresponding differential equation

Warmup #2:



Match the correct differential equation with the slope field shown above.

a. $\frac{dy}{dx} = x^2$ b. $\frac{dy}{dx} = y^2$ c. $\cancel{\frac{dy}{dx} = x^3}$ d. $\cancel{\frac{dy}{dx} = y^3}$ e. $\cancel{\frac{dy}{dx} = \frac{y}{x}}$

correct!

Then sketch a solution that passes through the indicated point.

Remember that the sketch of a solution on a slope field must meet the following requirements:

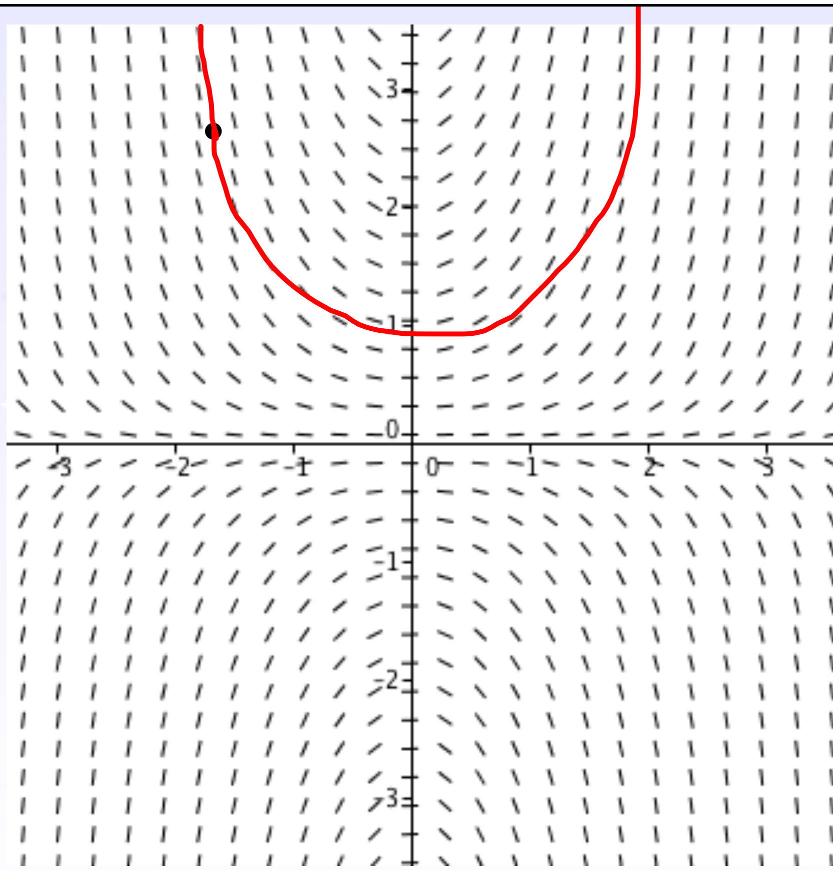
a) It must follow the trend of the slopes

b) It must pass through the indicated point

c) It must be a function

d) It must be drawn as far as possible within the domain and the given grid boundaries

Ditto!



a. $\cancel{\frac{dy}{dx} = x^2}$ b. $\frac{dy}{dx} = xy$ c. $\cancel{\frac{dy}{dx} = x^3}$ d. $\cancel{\frac{dy}{dx} = \frac{x}{y}}$ e. $\cancel{\frac{dy}{dx} = \ln x}$

✓ correct!

The domain of the solution to a differential equation must meet the following requirements:

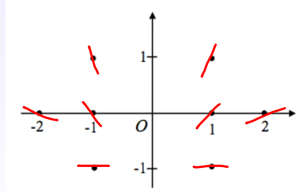
a) It must satisfy the domain of the solution

b) It must satisfy the domain of the original differential equation

c) It must be the largest <u>single</u> interval that contains the given point

Consider the differential equation dy/dx = (1 + y)/x, where x ≠ 0.

a) Sketch a slope field for the differential equation at the points indicated below.



b) Find the equation of the line tangent to f(x) at the point (-1, 1) and use it to approximate f(-1.1).

c) Find the particular solution, y = f(x), to the given differential equation with the initial condition, f(-1) = 1.

d) State the domain of the solution found in part c.

b) pt $(-1,1)$ $\quad y-1=-2(x+1)$

$m=-2$ $\quad y=-2(-1.1+1)+1$

$y=1.2$

$f(-1.1)\approx 1.2$

c) $\frac{dy}{dx}=\frac{1+y}{x}$

$\int\frac{1}{1+y}dy=\int\frac{1}{x}dx$

$\ln|1+y|=\ln|x|+C$

$|1+y|=e^{\ln|x|+C}=Ae^{\ln|x|}=A|x|$

$1+y=M|x|$

$y=M|x|-1$

$1=M-1,\ M=2$

$\boxed{y=2|x|-1}$

d) domain of $\frac{dy}{dx}=\frac{1+y}{x}$: $x\neq 0$

domain of y: $(-\infty,\infty)$

combined domains: $(-\infty,0)\cup(0,\infty)$

domain of solution: $(-\infty,0)$

Intersection

$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7\}$

union: $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

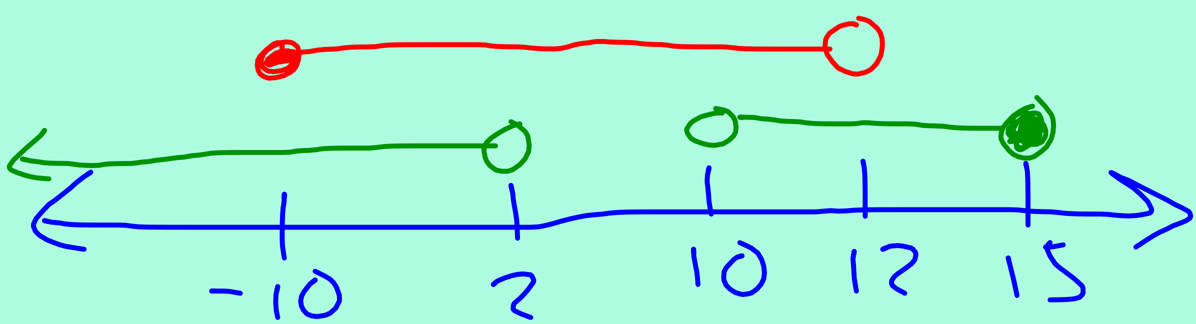
intersection:

$A \cap B = \{3, 4, 5\}$

$$A = (-\infty, 2) \cup (10, 15]$$

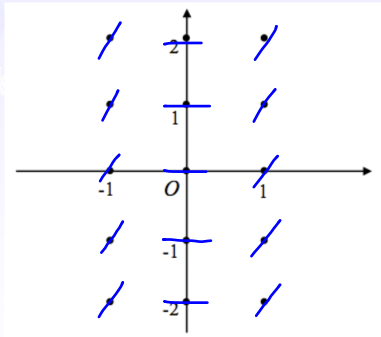
$$B = [-10, 12)$$

$$A \cap B = [-10, 2) \cup (10, 12)$$



Consider the differential equation $dy/dx = 3x^2$.

a) Sketch a slope field for the differential equation at the points indicated below.



b) Find the equation of the line tangent to f(x) at the point (1, 5) and use it to approximate f(1.2).

c) Find the particular solution, y = f(x), to the given differential equation with the initial condition, f(1) = 5.

d) State the domain of the solution found in part c.

b) pt (1,5) $y - 5 = 3(x-1)$
$m = 3$
$y = 3(1.2-1) + 5$
$f(1.2) \approx 5.6$

c) $\frac{dy}{dx} = 3x^2$ $\quad 5 = 1 + C$
$\int dy = \int 3x^2\,dx$ $\quad C = 4$
$y = x^3 + C$ $\quad y = x^3 + 4$

d) domain of $\frac{dy}{dx} = (-\infty, \infty)$
domain of $y = (-\infty, \infty)$
common domain $= (-\infty, \infty)$
domain of solution $= (-\infty, \infty)$

What have we learned?

- Can I sketch a slope field?
- Can I sketch the solution to a differential equation on a slope field?
- Can I match a slope field with its corresponding differential equation?

February 02, 2016