Warmup! Determine if the equation

$y'' + 2y' = 2e^{x}$ is true if $y = 2/3\,(e^{-2x} + e^{x})$

$$y' = \frac{2}{3}\left(-2e^{-2x} + e^{x}\right)$$

$$y'' = \frac{2}{3}\left(4e^{-2x} + e^{x}\right)$$

$$\frac{2}{3}\left(4e^{-2x} + e^{x}\right) + 2\left(\frac{2}{3}\right)\left(-2e^{-2x} + e^{x}\right) = 2e^{x}$$

$$\frac{8}{3}e^{-2x} + \frac{2}{3}e^{x} - \frac{8}{3}e^{-2x} + \frac{4}{3}e^{x} = 2e^{x}$$

$$2e^{x} = 2e^{x} \checkmark$$

6.1 and 6.3 Differential Equations!

Essential Learning Targets:

- > Uses antidifferentiation to find specific solutions to differential equations with given initial conditions
- > Solves differential equations by separation of variables
- > Knows that solutions to differential equations are functions or families of functions
- > Can find both a general solution to a differential equation as well as a particular solution satisfying a given initial condition.
- > States the restricted domain to the solution to a differential equation.

ex) Find the general solution to dy/dx = 2x/y .

$\frac{dy}{dx} = \frac{2x}{y}$

$\int y\,dy = \int 2x\,dx$

$\frac{1}{2}y^2 = x^2 + C$

$y^2 = 2x^2 + K$

$y = \pm\sqrt{2x^2 + K}$

Find the particular solution if y(1) = -3.

(x → 1, y → -3)

$-3 = \pm\sqrt{2 + K}$

$9 = 2 + K$

$K = 7$

$y = \pm\sqrt{2x^2 + 7}$

$y = -\sqrt{2x^2 + 7}$

The **domain of the particular solution** to a differential equation must meet the following requirements:

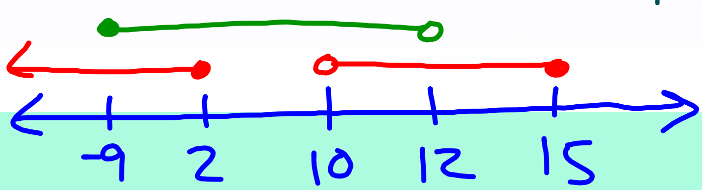
a) It must satisfy the domain of the particular solution

b) It must satisfy the domain of the original differential equation $\left(\frac{dy}{dx}\right)$

c) It must be the largest single interval that contains the given point

Think about it:

Suppose the domain of dy/dx is $(-\infty, 2] \cup (10, 15]$ and the domain of $y = f(x)$ is $[-9, 12)$. What would the domain of the solution be that contains a point at $x = 11$?



~~$[-9, 2]$~~ $\cup$ $(10, 12)$

Suppose xy' - 3y = 0.

Find the particular solution, y = f(x), if f(-3) = 2.

Write the solution in fully simplified form.

State the domain of this solution.

$(-3, 2)$

$x\frac{dy}{dx} - 3y = 0$

$x\frac{dy}{dx} = 3y$ $\quad \boxed{\frac{dy}{dx} = \frac{3y}{x}}$

$\int \frac{1}{y}\,dy = \int \frac{3}{x}\,dx$

$\ln|y| = 3\ln|x| + C$

$|y| = e^{3\ln|x| + C} = Ae^{3\ln|x|}$

$y = Me^{3\ln|x|}$

$2 = Me^{3\ln|-3|}$

$2 = Me^{3\ln 3}$

$2 = Me^{\ln 27}$

$2 = M \cdot 27$

$M = \frac{2}{27}$

$y = \frac{2}{27}e^{3\ln|x|}$

domain of $\frac{dy}{dx}$: $(-\infty, 0) \cup (0, \infty)$

domain of y: $(-\infty, 0) \cup (0, \infty)$

domain of solution: $\boxed{(-\infty, 0)}$ b/c $\underline{x = -3}$

Let's move!

(problems and solutions are on the next 4 screens)

1) Find a general solution to 2 + 3xyy' = 0. Then find the particular solution, y = f(x), that passes through the point (1, -2) and state its domain. (Warning, this domain is a bit tricky.) ☺

1) $\frac{dy}{dx} = \frac{-2}{3xy}$

$ydy = -\frac{2}{3} \cdot \frac{1}{x} dx$

$\frac{1}{2}y^2 = -\frac{2}{3}\ln|x| + C$

$y^2 = -\frac{4}{3}\ln|x| + K$

general solution: $\boldsymbol{y = \pm\sqrt{-\frac{4}{3}\ln|x| + K}}$

applying the point (1, -2) we get:

particular solution: $\boldsymbol{y = -\sqrt{-\frac{4}{3}\ln|x| + 4}}$

DOMAIN:

domain of $\frac{dy}{dx}$: $x \neq 0, y \neq 0$

domain of y: $-\frac{4}{3}\ln|x| + 4 \geq 0, x \neq 0$

solving for x we get: $x = \pm e^3$

make a sign line to see that domain of y: $[-e^3, 0) \cup (0, e^3]$

Domain of solution: $\boldsymbol{(0, e^3)}$

2) If the slope of the line tangent to f(x) is given by x(1 + y), solve for y = f(x) if f(0) = 5 and state the domain of the solution.

2) $\frac{dy}{dx} = x(1+y)$

$\frac{1}{1+y}dy = xdx$

$\ln|1+y| = \frac{1}{2}x^2 + C$

$|1+y| = Ae^{\frac{1}{2}x^2}$

general solution: $\boldsymbol{y = Me^{\frac{1}{2}x^2} - 1}$

Applying the point (0, 5) we get:

particular solution: $\boldsymbol{y = 6e^{\frac{1}{2}x^2} - 1}$

Domain of dy/dx: (-∞, ∞)

Domain of y: (-∞, ∞)

Domain of solution: $(-\infty, \infty)$

3) Suppose $2x + 3yy' = 0$. Find the solution $y = f(x)$ if $y = -2$ when $x = 1$. Then state the domain of the solution.

3) $\frac{dy}{dx} = -\frac{2x}{3y}$

$ydy = -\frac{2}{3}xdx$

$\frac{1}{2}y^2 = -\frac{1}{3}x^2 + C$

$y^2 = -\frac{2}{3}x^2 + K$

general solution: $\boldsymbol{y = \pm\sqrt{-\frac{2}{3}x^2 + K}}$

applying the initial condition: (1, -2)

particular solution: $\boldsymbol{y = -\sqrt{-\frac{2}{3}x^2 + \frac{14}{3}}}$

domain of dy/dx: $y \neq 0$

domain of y: $-\frac{2}{3}x^2 + \frac{14}{3} \geq 0$

$x = \pm\sqrt{7}$

making a sign line we get: $[-\sqrt{7}, \sqrt{7}]$

Domain of solution: $\boldsymbol{(-\sqrt{7}, \sqrt{7})}$

4) $\frac{dy}{dx} = \frac{\cos(\pi x)}{y}$

Solve this for y = f(x) given an initial condition of f(1) = 2 and state the domain of the solution.

4) $\frac{dy}{dx} = \frac{\cos(\pi x)}{y}$

$ydy = \cos(\pi x)\,dx$

$\frac{1}{2}y^2 = \frac{\sin(\pi x)}{\pi} + C$

$y^2 = \frac{2\sin(\pi x)}{\pi} + K$

general solution: $\boldsymbol{y = \pm\sqrt{\frac{2\sin(\pi x)}{\pi} + K}}$

Applying the point (1, 2) we get:

particular solution: $\boldsymbol{y = \sqrt{\frac{2\sin(\pi x)}{\pi} + 4}}$

domain of dy/dx: $y \neq 0$

domain of y: $(-\infty, \infty)$ (remember that $\sin(\pi x)$ will always be between -1 and 1

domain of solution: $(-\infty, \infty)$

Let's see an AP example!

2013 #6 (no calculator)

What have we learned?

- Can I verify solutions to a differential equation?
- Do I know the difference between general and particular solutions?
- Can I separate variables in order to solve a differential equation?
- Can I state the domain of the solution to a differential equation?