REVIEW DAY WARMUP!

1) $\int \dfrac{e^{2x} + 5}{e^{3x}}\, dx = \int \left(e^{-x} + 5e^{-3x}\right) dx$

$= -1e^{-x} - \dfrac{5}{3}e^{-3x} + C$

2) $\int 7(5^{(9-2x)})\, dx =$

$= -\dfrac{7}{2} \cdot \dfrac{5^{(9-2x)}}{\ln 5} + C$

3) d/dx arccos(5x) =

$= -\dfrac{5}{\sqrt{1-25x^2}}$

Let's do some review!!

Match each integral with its antiderivative.

1) $\int \frac{5}{9x^2+7}\,dx =$

2) $\int \frac{5x}{9x^2+7}\,dx =$

3) $\int \frac{5x^2}{9x^2+7}\,dx =$

4) $\int \frac{5}{x^2+4x+7}\,dx =$

5) $\int \frac{5x}{x^2+4x+7}\,dx =$

A $\frac{5}{2}\ln|x^2+4x+7| - \frac{10}{\sqrt{3}}\arctan\frac{x+2}{\sqrt{3}} + C$

B $\frac{5}{3\sqrt{7}}\arctan\frac{3x}{\sqrt{7}} + C$

C $\frac{5}{\sqrt{3}}\arctan\frac{x+2}{\sqrt{3}} + C$

D $\frac{5}{9}x - \frac{35}{27\sqrt{7}}\arctan\frac{3x}{\sqrt{7}} + C$

E $\frac{5}{18}\ln(9x^2+7) + C$

CONSTANT / X² + CONSTANT – inverse trig (u-sub if necessary)

1) $\int \frac{5}{9x^2+7}dx =$

let u = 3x, so du = 3dx and 1/3 du = dx

$$\frac{5}{3}\int \frac{1}{u^2+7}du = \frac{5}{3}\cdot\frac{1}{\sqrt{7}}\arctan\frac{u}{\sqrt{7}} + C = \frac{5}{3\sqrt{7}}\arctan\frac{3x}{\sqrt{7}} + C$$

$$\int \frac{5}{9}\cdot\frac{1}{x^2+\frac{7}{9}}dx = \frac{5}{9}\cdot\sqrt{\frac{9}{7}}\arctan\sqrt{\frac{9}{7}}\,x + C$$

$+ C$

(² + CONSTANT – u-sub (answer will have a natural log)

2) $\int \frac{5x}{9x^2+7}dx =$

let u = 9x² + 7, so du = 18xdx
and 5/18 du = 5xdx

$$\frac{5}{18}\int \frac{1}{u}du = \frac{5}{18}\ln|u| + C = \frac{5}{18}\ln(9x^2+7) + C$$

X² / X² + CONSTANT – long division, possible combination with inverse trig

3) $\int \frac{5x^2}{9x^2+7}dx =$

do long division, then an arctangent u-sub (same as problem #1)

$$\int \left(\frac{5}{9} - \frac{35}{9}\cdot\frac{1}{9x^2+7}\right)dx = \frac{5}{9}x - \frac{35}{27\sqrt{7}}\arctan\frac{3x}{\sqrt{7}} + C$$

CONSTANT / X² + X + CONSTANT – complete the square – inverse trig

4) $\int \frac{5}{x^2+4x+7}dx =$

$$\int \frac{5}{x^2+4x+4-4+7}dx = \int \frac{5}{(x+2)^2+3}dx = \frac{5}{\sqrt{3}}\arctan\frac{x+2}{\sqrt{3}} + C$$

(/ X² + X + CONSTANT – u-sub - complete the square – inverse trig

5) $\int \frac{5x}{x^2+4x+7}dx =$

let u = x² + 4x + 7, so du = (2x + 4)dx
so 5/2du = (5x + 10)dx

$$\int \frac{5x+10}{x^2+4x+7}dx + \int \frac{-10}{x^2+4x+7}dx$$

$$= \int \frac{5}{2}\cdot\frac{1}{u}du - \frac{10}{\sqrt{3}}\arctan\frac{x+2}{\sqrt{3}} + C$$

$$= \frac{5}{2}\ln|x^2+4x+7| - \frac{10}{\sqrt{3}}\arctan\frac{x+2}{\sqrt{3}} + C$$

Suppose $f(x) = x^3 - 3x^2 + 5x + 2$ and $g(x) = f^{-1}(x)$ (the inverse function of $f(x)$).

Find $g'(5)$ if $g(5) = 1$.

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$f'(x) = 3x^2 - 6x + 5$$

$$g'(5) = \frac{1}{f'(g(5))}$$

$$= \frac{1}{f'(1)}$$

$$= \frac{1}{2}$$

Match the differential equation to the slope field. Then sketch the solution that passes through the given point.

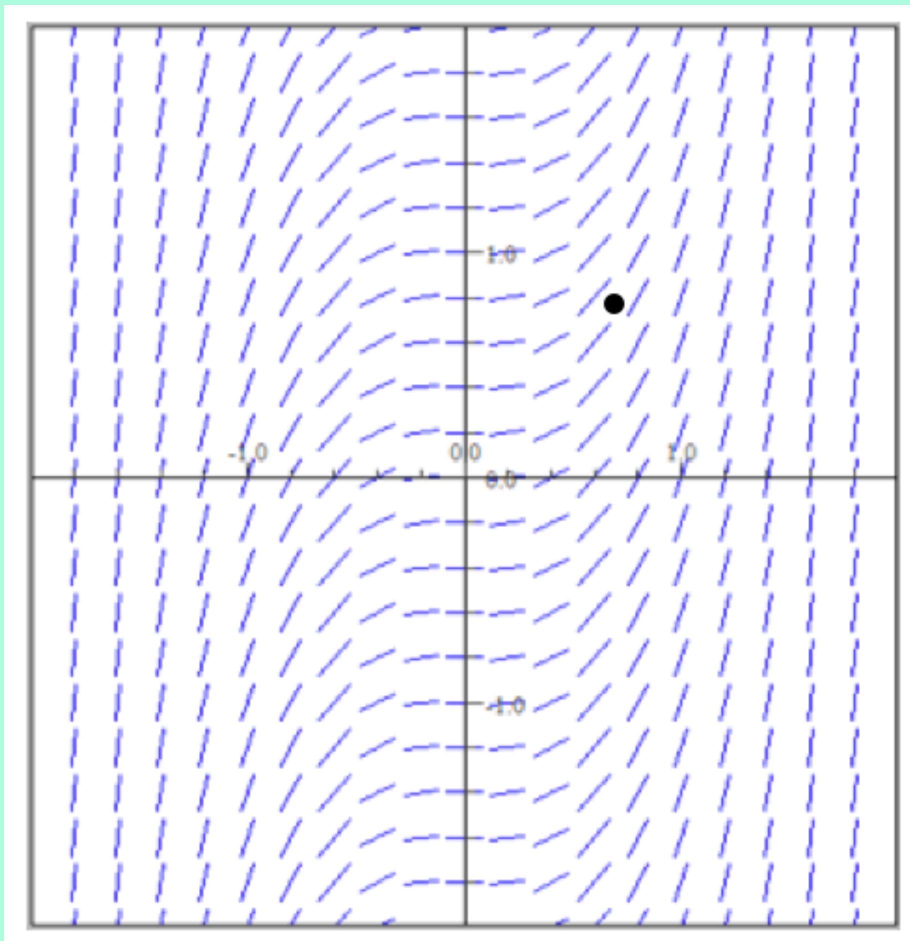
a) $\dfrac{dy}{dx} = x^3$

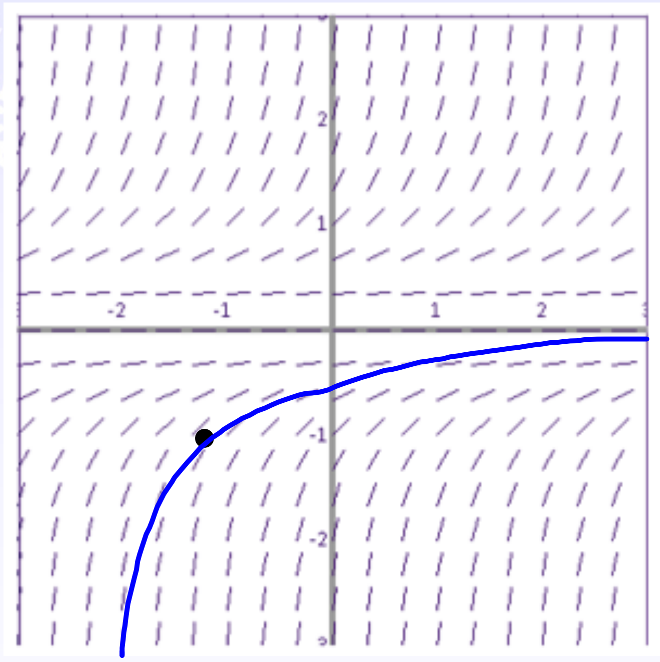
b) $\dfrac{dy}{dx} = 3x^2$

c) $\dfrac{dy}{dx} = 2x + y$

d) $\dfrac{dy}{dx} = \dfrac{x}{y}$

e) $\dfrac{dy}{dx} = \ln x$





a. $\frac{dy}{dx} = x^2$ b. $\frac{dy}{dx} = y^2$ $\cancel{\text{c.}}$ $\frac{dy}{dx} = x^3$ $\cancel{\text{d.}}$ $\frac{dy}{dx} = y^3$ $\cancel{\text{e.}}$ $\frac{dy}{dx} = \frac{y}{x}$

correct!

Then sketch a solution that passes through the indicated point.

Newton's law of cooling states that the rate at which an object changes temperature is proportional to the difference between its temperature and that of the surrounding medium.

If an object is in air of temperature 35° and the object cools from 120° to 60° in 40 minutes, find the temperature of the object after 100 minutes.

$$\frac{dP}{dt} = k(P-35)$$

$$\int \frac{1}{P-35}\,dP = \int k\,dt$$

$$\ln|P-35| = kt + C$$

$$P-35 = e^{kt+C} = Ae^{kt}$$

$$P = Ae^{kt} + 35$$

$$120 = Ae^{0} + 35$$

so $A = 85$

$$P = 85e^{kt} + 35$$

$$60 = 85e^{40k} + 35$$

$$\frac{25}{85} = e^{40k} \text{ so } k = \frac{1}{40}\ln\left(\frac{5}{17}\right)$$

$$P = 85e^{\frac{1}{40}\ln\left(\frac{5}{17}\right)t} + 35$$

$$P(100) = 85e^{\frac{1}{40}\ln\left(\frac{5}{17}\right)(100)} + 35$$

$$\approx 38.9876^{\circ}$$

1. $\int \frac{1}{3+25x^2}\,dx$ $= \frac{1}{5}\int \frac{1}{3+u^2}\,du$

$u=5x$

$du=5\,dx$ $= \frac{1}{5}\cdot\frac{1}{\sqrt{3}}\arctan\frac{u}{\sqrt{3}}+C$

$\frac{1}{5}du=dx$

$= \frac{1}{5\sqrt{3}}\arctan\frac{5x}{\sqrt{3}}+C$

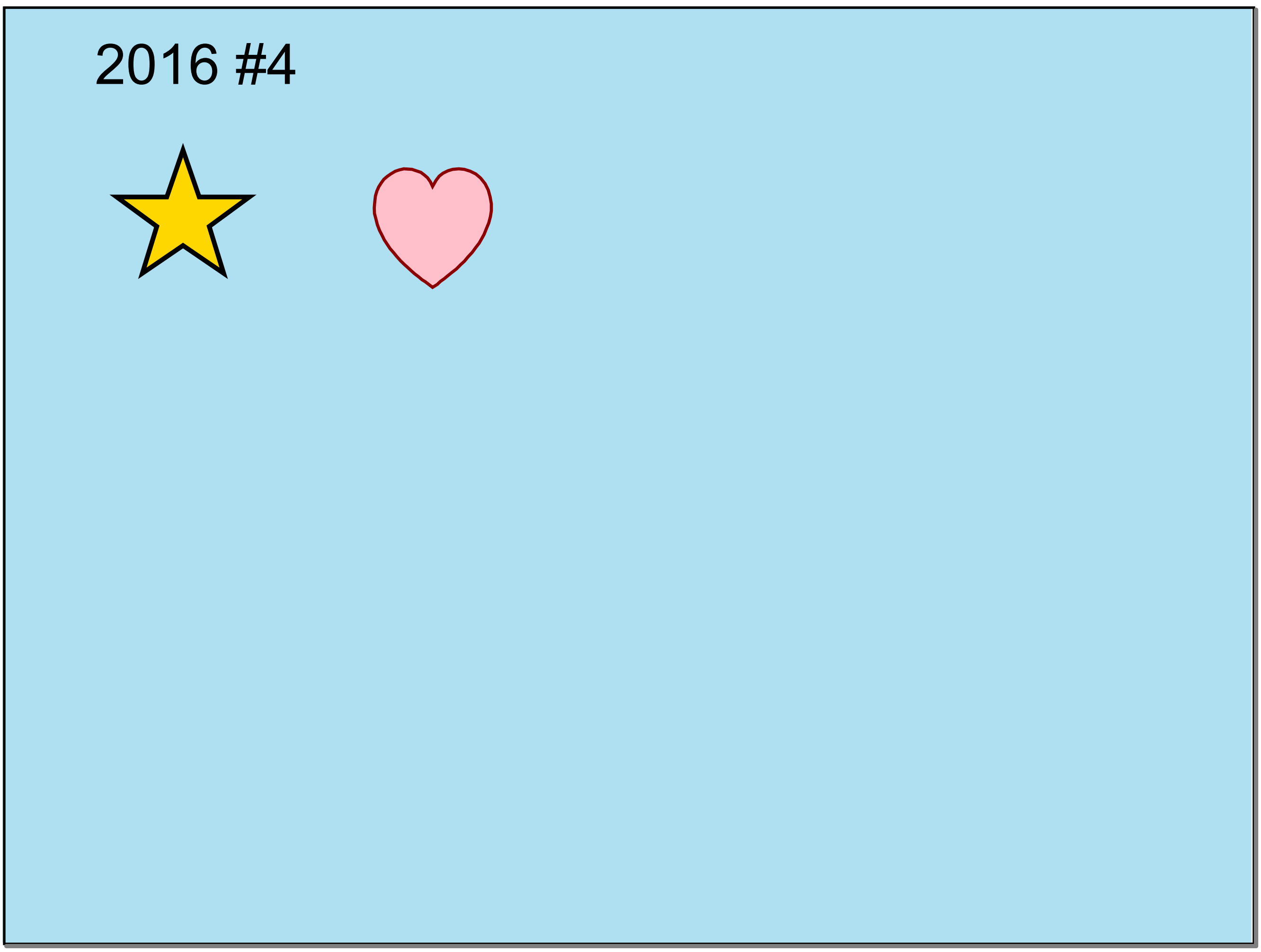
2. $\int \frac{e^{1/x}}{x^2}\,dx$

$= \int -e^u\,du$

$u=\frac{1}{x}$

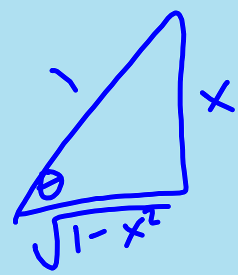
$du=-\frac{1}{x^2}dx$ $= -e^u + C$

$-du=\frac{1}{x^2}dx$ $= -e^{\frac{1}{x}}+C$


2016 #4

HW
pg 400

79 $y = \tan(\arcsin x)$



method 1

$$y' = \sec^2(\arcsin x)\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{1}{(1-x^2)\sqrt{1-x^2}}$$

method 2

$$y = \frac{x}{\sqrt{1-x^2}}$$

$$y' = \frac{\sqrt{1-x^2} - x\left(\cancel{\tfrac{1}{2}}\right)(1-x^2)^{-\frac{1}{2}}(-\cancel{2}x)}{1-x^2}$$

$$= \frac{\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}}{1-x^2} = \frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}} = \frac{1}{(1-x^2)\sqrt{1-x^2}}$$

83

$$y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2}\,\arcsin x$$

$$y' = (\arcsin x)^2 + x(2)(\arcsin x)\left(\frac{1}{\sqrt{1-x^2}}\right) - 2 + \cancel{2}\left(\cancel{\tfrac{1}{2}}\right)(1-x^2)^{-\frac{1}{2}}(-2x)(\arcsin x)$$

$$+\,2\cancel{\sqrt{1-x^2}}\left(\frac{1}{\cancel{\sqrt{1-x^2}}}\right)$$

$$= (\arcsin x)^2 + \frac{2x\arcsin x}{\sqrt{1-x^2}} - \frac{2x\arcsin x}{\sqrt{1-x^2}}$$

$$= (\arcsin x)^2$$

Quiz 5.5

#5

$$\frac{dy}{dx} = \frac{1}{xy}$$

$$y = -\sqrt{2\ln|x| + 16}$$

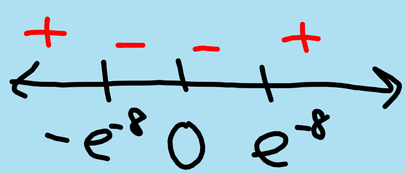
domain of $\frac{dy}{dx}$: $x \neq 0,\ y \neq 0$

domain of y: $2\ln|x| + 16 \geq 0$ $\quad (-\infty, -e^{-8}] \cup [e^{-8}, \infty)$

$\ln|x| = -8$

$|x| = e^{-8}$

$x = \pm e^{-8}$



$$\boxed{\text{domain: } (-\infty, -e^{-8})}$$