Let's Get Those Brains Moving...

$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

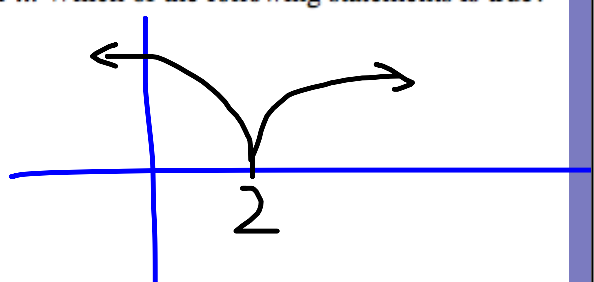
(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.



12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^4 e^u\,du$ (C) $2\int_1^2 e^u\,du$ (D) $\frac{1}{2}\int_1^2 e^u\,du$ (E) $\int_1^4 e^u\,du$

$du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx$

$2du = \frac{1}{\sqrt{x}}dx$

WARMUP!!

Can you figure this out?

(Hint, try factoring out a 1/9 first)

$$\int \frac{1}{x^2+9}\,dx = \frac{1}{9}\int \frac{1}{\frac{x^2}{9}+1}\,dx = \frac{1}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$$

$$\int \frac{1}{9}\cdot\frac{1}{\frac{x^2}{9}+1}\,dx = \int \frac{1}{9}\cdot\frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$$

✓ Let u = x/3, so du = 1/3 dx and 3 du = dx

$$\int \frac{1}{3}\cdot\frac{1}{u^2+1}\,du = \frac{1}{3}\arctan u + C = \frac{1}{3}\arctan\frac{x}{3} + C$$

5.7 Integrals involving Inverse Trig Functions!

At the end of this lesson you will be able to:

> Integrate functions where the result is an inverse trig function

> Identify when an integral is inverse trig vs power rule vs natural log rule

The Big Three!

1. $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

2. $$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

3. $$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|x|}{a} + C$$

ex) $\int \frac{1}{4+(x-1)^2}\,dx = \int \frac{1}{4+u^2}\,du$

$a^2 = 4$

$a = 2$

$u = x - 1$

$du = dx$

$= \frac{1}{2}\arctan\frac{u}{2} + C$

$= \frac{1}{2}\arctan\frac{x-1}{2} + C$

You try! (Hint: there are several ways to approach this problem.)

$$\int \frac{1}{\sqrt{7-16x^2}}\,dx = \int \frac{1}{\sqrt{7-(4x)^2}}\,dx$$

$a^2 = 7$

$a = \sqrt{7}$

$u = 4x$

$du = 4\,dx$

$\frac{1}{4}du = dx$

$$= \frac{1}{4}\int \frac{1}{\sqrt{7-u^2}}\,du$$

$$= \frac{1}{4}\arcsin\frac{u}{\sqrt{7}} + C$$

$$= \frac{1}{4}\arcsin\frac{4x}{\sqrt{7}} + C$$

Any ideas on how to approach this one?

Ex) $\int \frac{dx}{x^2 - 6x + 13} =$

$$= \int \frac{1}{x^2 - 6x + 9 + 13 - 9}\,dx$$

$\div 2$ ↓ $(-3)^2 \to 9$

$$= \int \frac{1}{(x-3)^2 + 4}\,dx = \frac{1}{2}\arctan\frac{x-3}{2} + C$$

Hmmmm. Try doing some u-sub first on this one to see if it can help guide you to an approach.

$$\int \frac{2x-5}{x^2+2x+2}dx = \int \frac{2x+2-7}{x^2+2x+2}dx$$

$u = x^2+2x+2$

$du = (2x+2)\,dx$

$$= \int \frac{2x+2}{x^2+2x+2}dx - 7\int \frac{1}{x^2+2x+2}dx$$

$$= \int \frac{1}{u}du - 7\int \frac{1}{x^2+2x+1+2-1}dx$$

$$= \ln|u| - 7\int \frac{1}{(x+1)^2+1}dx$$

$$= \ln|x^2+2x+2| - 7\arctan(x+1) + C$$

In general:

$\int \# / (x^2 + \#)$ -> arctangent (possible u-sub)

$\int x / (x^2 + \#)$ -> u-sub (answer will be natural log)

$\int x^2$ or greater $/ (x^2 + \#)$ -> long division (possible combination problem)

$\int \# / (x^2 + x + \#)$ -> arctangent - complete the square

$\int x / (x^2 + x + \#)$ -> u-sub (will probably need to split the integral)

$\int x^2$ or greater $/ (x^2 + x + \#)$ -> long division (possible combo problem)

Let's do some review!

1) $\int \frac{6x}{2x+3}\,dx =$

2) $\int 7(2^{5x})dx =$

3) $\int \frac{3}{e^{4x}}\,dx =$

4) $\int 6\sec(2x)\,dx =$

5) $\int \frac{2}{x^2+6x+23}\,dx =$

6) $\int \frac{7x}{x^2+2x+9}\,dx =$

7) $\int \frac{3\sin^3 x}{\sec x}\,dx =$

1) $3x - 9\ln|2x+3| + C$

2) $\frac{7(2^{5x})}{5\ln 2} + C$

3) $\frac{-3}{4e^{4x}} + C$ ✓

4) $3\ln|\sec 2x + \tan 2x| + C$

5) $\frac{2}{\sqrt{14}}\arctan\frac{x+3}{\sqrt{14}} + C$

6) $\frac{7}{2}\ln|x^2+2x+9| - \frac{7}{\sqrt{8}}\arctan\frac{x+1}{\sqrt{8}} + C$

7) $\frac{3}{4}\sin^4 x + C$

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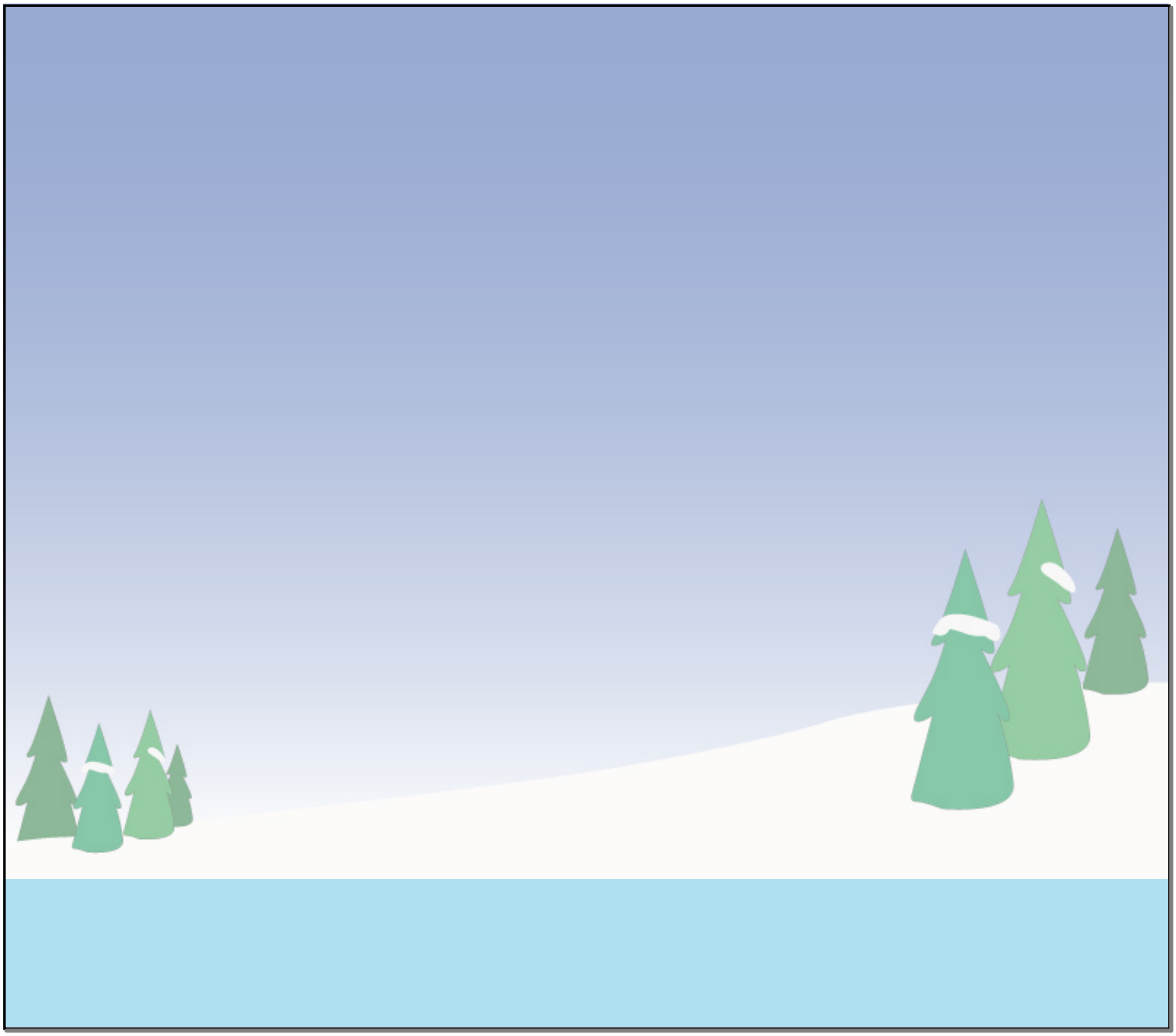

What have we learned?

- Can I evaluate integrals where the results are inverse trig functions?
- Can I recognize when an integral is inverse trig vs natural log?

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