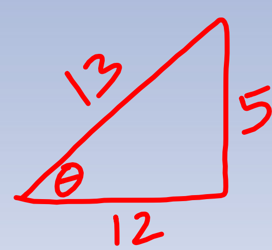
Warmup! Simplify the following.

Hint: if you get stuck, draw a triangle.

1) $\tan(\arccos\sqrt{2} / 2)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

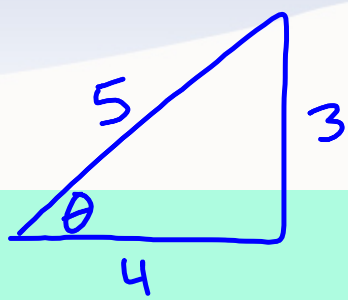



2) $\cos(\arcsin 5/13)$

$\cos(\theta) = \frac{12}{13}$

3) $\sec(\arctan(-3/4))$

$\sec(\theta) = \frac{5}{4}$



59 $y=(\ln x)^{\cos x}$ $(e,1)$

$$\ln y = \ln\left((\ln x)^{\cos x}\right)$$

$$\ln y = \cos x\left[\ln(\ln x)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x\left[\ln(\ln x)\right] + \cos x \frac{\frac{1}{x}}{\ln x}$$

$$\frac{dy}{dx} = \left[-\sin x(\ln(\ln x)) + \frac{\cos x}{x\ln x}\right](\ln x)^{\cos x}$$

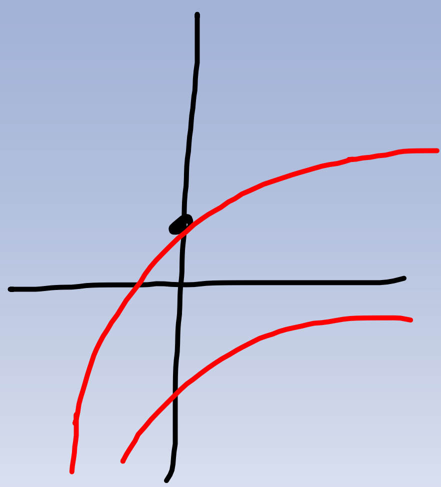
$$\left.\frac{dy}{dx}\right|_{x=e} = \left[-\sin e(0) + \frac{\cos e}{e}\right](1)^{\cos e}$$

$$= \frac{\cos e}{e}$$

$$y-1 = \frac{\cos e}{e}(x-e)$$

(13) $\frac{dy}{dx} = .4^{\frac{x}{3}}$ $\quad (0, \frac{1}{2})$

$$\int dy = \int .4^{\frac{x}{3}}\, dx$$



$$y = 3\int .4^{u}\, du$$

$$y = \frac{3(.4^{\frac{x}{3}})}{\ln(.4)} + C$$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3du = dx$

$$\frac{1}{2} = \frac{3(1)}{\ln .4} + C$$

$$C = \frac{1}{2} - \frac{3}{\ln .4}$$

$$y = \frac{3(.4^{\frac{1}{3}x})}{\ln .4} + \frac{1}{2} - \frac{3}{\ln .4}$$

79 $C(t) = P(1.05)^t$

b) $C'(t) = P(1.05)^t(\ln 1.05)$

$\cancel{P}\cancel{(1.05)^t}(\ln 1.05) = K\cancel{P}\cancel{(1.05)^t}$

$\underline{\ln 1.05 = K}$

91 $V = 6.7e^{\left(\frac{-48.1}{t}\right)}$

a) $\lim_{t \to \infty} 6.7e^{\frac{-48.1}{t}}$

$= \lim_{t \to \infty} \frac{6.7}{e^{\frac{48.1}{t}}} = 6.7$

b) $V' = 6.7e^{\left(-\frac{48.1}{t}\right)}\left(\frac{48.1}{t^2}\right)$

# 5.6 Derivatives of Inverse Trig Functions!

Essential Learning Target:

> Knows the specific rules for calculating the derivatives of inverse trigonometric functions

if $\sin\frac{\pi}{6} = \frac{1}{2}$ then $\arcsin\frac{1}{2} = \frac{\pi}{6}$

input is an angle

output is a ratio

input is a ratio

output is an angle

Domain of inverse trig functions

y = arcsinx

y = arctanx

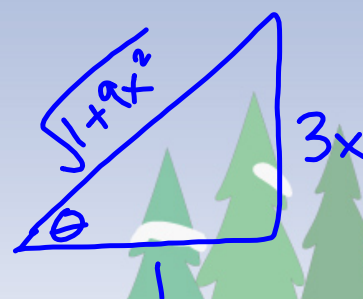
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

y = arccosx

[0, π]

Rewrite sec(arctan(3x)) in algebraic form.

$$\sec(\arctan(3x)) = \sqrt{1+9x^2}$$



Let's derive a derivative!

ex) If y = arcsinx, find y'

$$\sin y = \sin(\arcsin x)$$

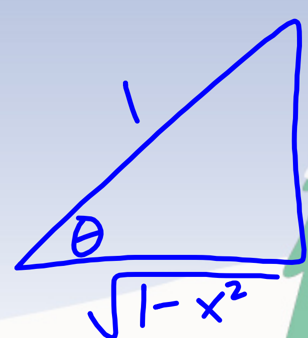
$$\sin y = x$$

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \sec y$$

$$\frac{dy}{dx} = \sec(\arcsin x)$$

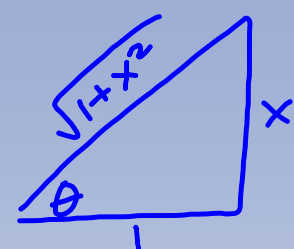
$$= \frac{1}{\sqrt{1-x^2}}$$



You try! If y = arctanx, find y'

$\tan y = x$

$\sec^2 y \frac{dy}{dx} = 1$




$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \cos^2(\arctan x)$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

# Derivatives of all 6 inverse trig functions!

## Yay, more memorizing!!

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}\operatorname{arccot} u = -\frac{u'}{1+u^2}$$

$$\frac{d}{dx}\operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}\operatorname{arccsc} u = -\frac{u'}{|u|\sqrt{u^2-1}}$$

1) $-\frac{4^{-x}}{ln\,4} + C$

2) $-e^{1-x} + C$

3) $-\frac{7^{(3-x)^2}}{2ln7} + C$

4) $\ln|e^x + e^{-x}| + C$

5) $\frac{\ln|2+5^{4x}|}{4\ln 5} + C$

6) $x^2 - x + C$

7) $\frac{2^{\sin x}}{\ln 2} + \mathrm{C}$

8) $-2\ln\left|\csc\left(\frac{x}{2}\right) + \cot\left(\frac{x}{2}\right)\right| + C$

9) $-\frac{1}{3}\ln|6-x^3| + C$

10) $\ln|\ln x| + C$

ex) Use your newly learned formulas to differentiate
$y = 2\text{arcsec}(3x^2)$

$$y' = 2 \cdot \frac{6x}{|3x^2|\sqrt{(3x^2)^2 - 1}}$$

$$= \frac{4}{x\sqrt{9x^4 - 1}}$$

note: $x / |x^2| = 1/x$, not $1/|x|$

You try! Find y' for each of the following:

1) $y = \text{arccot}(5x^3)$

2) $y = 8\arccos(2x)$

3) $y = x\,\text{arcsec}(6x^2)$

1) $y' = -\dfrac{15x^2}{1 + 25x^6}$

2) $y' = -\dfrac{16}{\sqrt{1 - 4x^2}}$

3) $y' = \text{arcsec}(6x^2) + \dfrac{2}{\sqrt{36x^4 - 1}}$

If you've got the time, I've got the integrals!

Give these a try!

1) $\int \sin(4x)\,dx$

2) $\int e^{2x}dx$

3) $\int (2x+1)^4 dx$

4) $\int \tan(5x)\,dx$

5) $\int e^x(1+e^x)^6 dx$

6) $\int \frac{1}{2x+1}dx$

7) $\int \csc(\pi x)\,dx$

8) $\int \tan^2 x \sec^2 x\,dx$

9) $\int \frac{e^x}{3+e^x}dx$

10) $\int \frac{\ln x}{x}dx$

1) $-\frac{1}{4}\cos(4x)+C$

2) $\frac{1}{2}e^{2x}+C$

3) $\frac{1}{10}(2x+1)^5+C$

4) $-\frac{1}{5}\ln|\cos(5x)|+C$

5) $\frac{1}{7}(1+e^x)^7+C$

6) $\frac{1}{2}\ln|2x+1|+C$

7) $-\frac{1}{\pi}\ln|\csc(\pi x)+\cot(\pi x)|+C$

8) $\frac{1}{3}\tan^3 x+C$

9) $\ln|3+e^x|+C$

10) $\frac{1}{2}(\ln x)^2+C$

# Review!!

Find dy/dx for each of the following:

1) $y = 3x^2$

2) $y = e^{3x^\wedge 2}$

3) $y = \cot(3x^2)$

4) $y = \ln(3x^2)$

5) $y = (e^{3x^\wedge 2})^2$

6) $y = \sin(5x)$

7) $y = \sin^3(5x)$

8) $y = e^{\tan x}$

9) $y = (1 + e^x)^5$

10) $y = \ln(\sin(7x))$

✓

1) $6x$

2) $6xe^{3x^\wedge 2}$

3) $-6x\csc^2(3x^2)$

4) $2/x$

5) $12xe^{6x^\wedge 2}$

6) $5\cos(5x)$

7) $15\sin^2(5x)\cos(5x)$

8) $\sec^2 x \cdot e^{\tan x}$

9) $5e^x(1 + e^x)^4$

10) $7\cot(7x)$

# What have we learned?

- Can I state the derivative formulas for all 6 inverse trig functions and apply them correctly?