WARMUP!!

Find the derivative of $y = x^{(2x - 3)}$

(Hint, use logarithmic differentiation)

$$\ln y = \ln x^{(2x-3)}$$

$$\ln y = (2x-3)\cdot \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x + (2x-3)\cdot\frac{1}{x}$$

$$= \left[2\ln x + \frac{2x-3}{x}\right]\cdot x^{(2x-3)}$$

✓ $dy/dx = (2\ln x + 2 - 3/x) \cdot x^{(2x - 3)}$

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$$g(t)=\frac{10\log_4 t}{t}$$

$$g'(t)=\frac{10\cdot\frac{1}{\cancel{t}\ln 4}\cdot\cancel{t}-10\log_4 t}{t^2}$$

$$=\frac{\frac{10}{\ln 4}-\frac{10\ln t}{\ln 4}}{t^2}=\frac{10-10\ln t}{t^2\ln 4}$$

5.5b Integrals with Other Bases!

At the end of this lesson you will be able to:

> Integrate logarithmic and exponential functions with bases other than e

Fill in the formula! Think about reversing the derivative to see if you can fill this formula in.

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

Ex) $\int 2^x\,dx = \frac{2^x}{\ln 2} + C$

Ex) $\int 2^{3x}\,dx = \frac{2^{3x}}{3\ln 2} + C$

You try!

$$\int_0^1 (x+3)2^{(x+3)^2}\,dx = \frac{1}{2}\int_9^{16} 2^u\,du$$

$u = (x+3)^2$

$du = 2(x+3)\,dx$

$\frac{1}{2}du = (x+3)dx$

$$= \frac{1}{2}\cdot\frac{2^u}{\ln 2}\Big|_9^{16}$$

$$= \frac{2^{16}-2^9}{2\ln 2} = \frac{2^{15}-2^8}{\ln 2}$$

Let's do some review! Can you integrate the following in 15 minutes or less?

1) $\int 4^{-x}dx$

2) $\int e^{1-x}dx$

3) $\int (3-x)7^{(3-x)^2}\,dx$

4) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

5) $\int \frac{5^{4x}}{2+5^{4x}}dx$

6) $\int \ln(e^{2x-1})dx$

7) $\int 2^{\sin x}\cos x\,dx$

8) $\int \csc\left(\frac{x}{2}\right)dx$

9) $\int \frac{x^2}{6-x^3}dx$

10) $\int \frac{1}{x\ln x}dx$

1) $-\frac{4^{-x}}{ln\,4}+C$ 2) $-e^{1-x}+C$ 3) $-\frac{7^{(3-x)^2}}{2ln7}+C$

4) $\ln|e^x+e^{-x}|+C$ 5) $\frac{\ln|2+5^{4x}|}{4\ln 5}+C$ 6) x^2-x+C

7) $\frac{2^{\sin x}}{\ln 2}+\mathrm{C}$ 8) $-2\ln\left|\csc\left(\frac{x}{2}\right)+\cot\left(\frac{x}{2}\right)\right|+C$

9) $-\frac{1}{3}\ln|6-x^3|+C$ 10) $\ln|\ln x|+C$

(4) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx = \int \frac{1}{u}\, du$

$= \ln|u| + C$

$= \ln(e^x + e^{-x}) + C$

$u = e^x + e^{-x}$

$du = (e^x - e^{-x})\, dx$

(6) $\int \ln e^{2x-1}\, dx = \int (2x-1)\, dx$

$= x^2 - x + C$

(8) $\int \csc\left(\frac{x}{2}\right) dx = \int 2\csc u\, du$

$u = \frac{x}{2}$

$du = \frac{1}{2}\, dx$

$2\, du = dx$

$= -2\ln|\csc u + \cot u| + C$

$= -2\ln\left|\csc \frac{x}{2} + \cot \frac{x}{2}\right| + C$

(7) $\int 2^{\sin x} \cos x\, dx = \int 2^u\, du$

$u = \sin x$

$du = \cos x\, dx$

$= \frac{2^u}{\ln 2} + C$

$= \frac{2^{\sin x}}{\ln 2} + C$

(5) $\int \frac{5^{4x}}{2 + 5^{4x}}\, dx = \int \frac{1}{4\ln 5} \cdot \frac{1}{u}\, du$

$u = 2 + 5^{4x}$

$du = 5^{4x}(4)(\ln 5)\, dx$

$\frac{1}{4\ln 5}\, du = 5^{4x}\, dx$

$= \frac{1}{4\ln 5} \cdot \ln|u| + C$

$= \frac{\ln(2 + 5^{4x})}{4\ln 5} + C$

What have we learned?

- Can I integrate a function involving a^x?
- Can I solve interest related problems?