

WARMUP!!

Find the derivative of $y = x^{(2x-3)}$

(Hint, use logarithmic differentiation)

$$\ln y = \ln x^{(2x-3)}$$

$$\ln y = (2x-3) \cdot \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = 2 \ln x + (2x-3) \cdot \frac{1}{x}$$
$$= \left[2 \ln x + \frac{2x-3}{x} \right] \cdot x^{(2x-3)}$$

✓ $dy/dx = (2 \ln x + 2 - 3/x) \cdot x^{(2x-3)}$

$$(47) \quad g(t) = \frac{10 \log_4 t}{t}$$

$$g'(t) = \frac{10 \cdot \frac{1}{t \ln 4} \cdot t - 10 \log_4 t}{t^2}$$

$$= \frac{\frac{10}{\ln 4} - \frac{10 \ln t}{\ln 4}}{t^2} = \frac{10 - 10 \ln t}{t^2 \ln 4}$$

5.5b Integrals with Other Bases!

At the end of this lesson you will be able to:

- > Integrate logarithmic and exponential functions with bases other than e

Fill in the formula! Think about reversing the derivative to see if you can fill this formula in.

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\text{Ex) } \int 2^x dx = \frac{2^x}{\ln 2} + C$$

$$\text{Ex) } \int 2^{3x} dx = \frac{2^{3x}}{3 \ln 2} + C$$



You try!

$$\int_0^1 (x+3)2^{(x+3)^2} dx$$

$$= \frac{1}{2} \int_9^{16} 2^u du$$

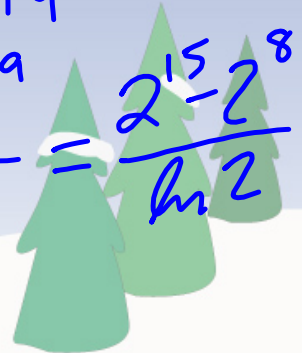
$$u = (x+3)^2$$

$$du = 2(x+3) dx$$

$$\frac{1}{2} du = (x+3) dx$$

$$= \frac{1}{2} \cdot \frac{2^u}{\ln 2} \Big|_9^{16}$$

$$= \frac{2^{16} - 2^9}{2 \ln 2} = \frac{2^{15} - 2^8}{\ln 2}$$



Let's do some review! Can you integrate the following in 15 minutes or less?

$$1) \int 4^{-x} dx$$

$$2) \int e^{1-x} dx$$

$$3) \int (3-x)7^{(3-x)^2} dx$$

$$4) \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$5) \int \frac{5^{4x}}{2 + 5^{4x}} dx$$

$$6) \int \ln(e^{2x-1}) dx$$

$$7) \int 2^{\sin x} \cos x dx$$

$$8) \int \csc\left(\frac{x}{2}\right) dx$$

$$9) \int \frac{x^2}{6-x^3} dx$$

$$10) \int \frac{1}{x \ln x} dx$$

$$1) -\frac{4^{-x}}{\ln 4} + C$$

$$2) -e^{1-x} + C$$

$$3) -\frac{7^{(3-x)^2}}{2 \ln 7} + C$$

$$4) \ln|e^x + e^{-x}| + C$$

$$5) \frac{\ln|2 + 5^{4x}|}{4 \ln 5} + C$$

$$6) x^2 - x + C$$

$$7) \frac{2^{\sin x}}{\ln 2} + C$$

$$8) -2 \ln \left| \csc\left(\frac{x}{2}\right) + \cot\left(\frac{x}{2}\right) \right| + C$$

$$9) -\frac{1}{3} \ln|6-x^3| + C$$

$$10) \ln |\ln x| + C$$

$$\textcircled{4} \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \int \frac{1}{u} du$$

$$= \ln|u| + C$$

$$= \ln(e^x + e^{-x}) + C$$

$$u = e^x + e^{-x}$$

$$du = (e^x - e^{-x}) dx$$

$$\textcircled{6} \int \ln e^{2x-1} dx = \int (2x-1) dx$$

$$= x^2 - x + C$$

$$\textcircled{8} \int \csc\left(\frac{x}{2}\right) dx = \int 2 \csc u du$$

$$u = \frac{x}{2}$$

$$du = \frac{1}{2} dx$$

$$2 du = dx$$

$$= -2 \ln|\csc u + \cot u| + C$$

$$= -2 \ln\left|\csc \frac{x}{2} + \cot \frac{x}{2}\right| + C$$

$$\textcircled{7} \int 2^{\sin x} \cos x dx = \int 2^u du$$

$$u = \sin x$$

$$du = \cos x dx$$

$$= \frac{2^u}{\ln 2} + C$$

$$= \frac{2^{\sin x}}{\ln 2} + C$$

$$\textcircled{5} \int \frac{5^{4x}}{2+5^{4x}} dx = \int \frac{1}{4 \ln 5} \cdot \frac{1}{u} du$$

$$u = 2 + 5^{4x}$$

$$du = 5^{4x} (4) (\ln 5) dx$$

$$\frac{1}{4 \ln 5} du = 5^{4x} dx$$

$$= \frac{1}{4 \ln 5} \cdot \ln|u| + C$$

$$= \frac{\ln(2+5^{4x})}{4 \ln 5} + C$$

What have we learned?

- Can I integrate a function involving a^x ?
- Can I solve interest related problems?