WARMUP!!

Integrate the following:

1) $\displaystyle\int \frac{1}{(2x-3)^5}\,dx = \int \frac{1}{2}\cdot u^{-5}\,du$

$u = 2x-3$

$du = 2\,dx$

$= -\frac{1}{8}u^{-4} + C$

2) $\displaystyle\int \frac{1}{2x-3}\,dx$

✓

1) $-\dfrac{1}{8(2x-3)^4} + C$

2) 1/2 ln|2x - 3| + C

61 $f(x)=(3+2x)e^{-3x}$

$$f'(x)=2e^{-3x}+(3+2x)(e^{-3x})(-3)$$

$$=2e^{-3x}-9e^{-3x}-6xe^{-3x}$$

$$=-7e^{-3x}-6xe^{-3x}$$

$$=-e^{-3x}(7+6x)$$

$$f''(x)=(-e^{-3x})(-3)(7+6x)-e^{-3x}(6)$$

$$=21e^{-3x}+18xe^{-3x}-6e^{-3x}$$

$$=15e^{-3x}+18xe^{-3x}$$

69 $f(x)=x^2e^{-x}$

$f'(x)=2xe^{-x}+x^2e^{-x}(-1)$

$=xe^{-x}(2-x)$

$x=0,2$

$$f' \quad \begin{array}{ccccc} - & & + & & - \\ \hline & 0 & & 2 & \\ \searrow & & \nearrow & & \searrow \end{array}$$

rel max @ $\left(2,\frac{4}{e^2}\right)$ b/c f' chgs from + to −

rel min @ $(0,0)$ b/c f' chgs from − to +

$f''(x)=2e^{-x}+2xe^{-x}(-1)-2xe^{-x}-x^2e^{-x}(-1)$

$=2e^{-x}-4xe^{-x}+x^2e^{-x}$

$=e^{-x}(x^2-4x+2)$

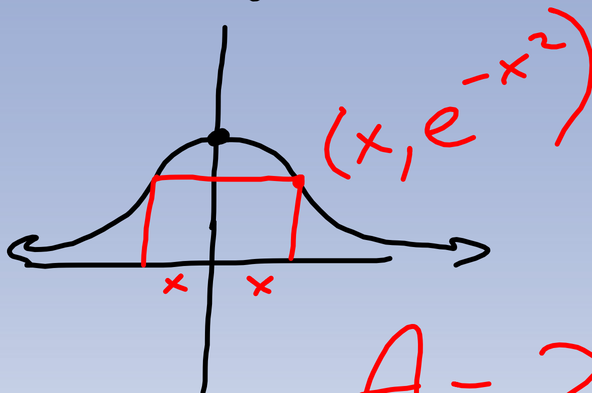
$x=\frac{4\pm\sqrt{16-8}}{2}=\frac{4\pm2\sqrt{2}}{2}=2\pm\sqrt{2}$

$$f'' \quad \begin{array}{ccccc} + & & - & & + \\ \hline & 2-\sqrt{2} & & 2+\sqrt{2} & \\ \cup & & \cap & & \cup \end{array}$$

poi @ $\left(2-\sqrt{2},(2-\sqrt{2})^2e^{-(2-\sqrt{2})}\right)$ and

$\left(2+\sqrt{2},(2+\sqrt{2})^2e^{-(2+\sqrt{2})}\right)$ b/c f'' chgs sign

73. $y=e^{-x^2}$



$$A = 2x e^{-x^2}$$

$$A' = 2e^{-x^2} + 2x e^{-x^2}(-2x)$$

$$= 2e^{-x^2}(1-2x^2) = 0$$

$$x^2 = \frac{1}{2} \qquad x = \pm\sqrt{\frac{1}{2}}$$

$$A = 2\sqrt{\frac{1}{2}}\, e^{-\frac{1}{2}}$$

97 $\int e^{-x}\tan(e^{-x})\,dx$

$u = e^{-x}$

$du = -e^{-x}\,dx$

$-du = e^{-x}\,dx$

$= \int -\tan u\,du$

$= \ln|\cos u| + C$

$= \ln|\cos e^{-x}| + C$

99 $$\int_0^1 e^{-2x}\,dx = \int_0^{-2} -\frac{1}{2}e^u\,du$$

$u = -2x$

$du = -2\,dx$

$-\frac{1}{2}du = dx$

$$= \int_{-2}^{0} \frac{1}{2}e^u\,du$$

$$= \frac{1}{2}e^u \Big|_{-2}^{0}$$

$$= \frac{1}{2} - \frac{1}{2}e^{-2}$$

$$= \frac{1}{2} - \frac{1}{2e^2} = \frac{e^2-1}{2e^2}$$

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$$\int_1^3 \frac{e^{3/x}}{x^2}\,dx = \int_3^1 -\frac{1}{3}e^u\,du$$

$$u = \frac{3}{x}$$

$$du = -3x^{-2}\,dx$$

$$du = \frac{-3}{x^2}\,dx$$

$$-\frac{1}{3}\,du = \frac{1}{x^2}\,dx$$

$$= \int_1^3 \frac{1}{3}e^u\,du$$

$$= \frac{1}{3}e^u \Big|_1^3$$

$$= \frac{1}{3}e^3 - \frac{1}{3}e$$

107 $\frac{dy}{dx} = xe^{ax^2}$

$\int dy = \int xe^{ax^2}\, dx$

$y = \int \frac{1}{2a} e^u\, du$

$u = ax^2$

$du = 2ax\, dx$

$\frac{1}{2a} du = x dx$

$y = \frac{1}{2a} e^u + C$

$y = \frac{e^{ax^2}}{2a} + C$

115 $y = xe^{-\frac{x^2}{4}}$, $y=0$, $x=0$, $x=\sqrt{6}$

$$\int_0^{\sqrt{6}} xe^{-\frac{x^2}{4}}\,dx = \int_0^{-\frac{3}{2}} -2e^u\,du$$

$u = -\frac{1}{4}x^2$

$du = -\frac{1}{2}x\,dx$

$-2du = x\,dx$

$$= \int_{-\frac{3}{2}}^{0} 2e^u\,du$$

$$= 2e^u \Big|_{-\frac{3}{2}}^{0}$$

$$= 2 - 2e^{-\frac{3}{2}}$$

5.5a Derivatives with Other Bases!

At the end of this lesson you will be able to:

> Differentiate logarithmic and exponential functions with bases other than e

Warmup #2: Fill in the blanks:

If $\log_a b = c$, then $a^{\_\_} =$ ___

Many logarithmic/exponential equations can be solved by switching between these two forms

Warmup #3: Solve the following for x:

1) $\log_x 25 = 2$

2) $\log_3(1/81) = x$

3) $5^{6x} = 8320$

4) $3^{5-x} = 80$

① $x^2 = 25$

$x = \cancel{\pm} 5$

② $3^x = \frac{1}{81}$

$x = -4$

③ $\log_5 8320 = 6x$

$x = \frac{\log_5 8320}{6}$

④ $\log_3 80 = 5 - x$

$x = 5 - \log_3 80$

Change of base formula: $\log_a x = \dfrac{\ln x}{\ln a}$

Can you use this info to answer the following?

Find the derivative of $y = \log_2(x^2 - 3)$

$$y = \frac{\ln(x^2-3)}{\ln 2} = \frac{1}{\ln 2} \cdot \ln(x^2-3)$$

$$y' = \frac{1}{\ln 2} \cdot \frac{2x}{x^2-3}$$

$$= \frac{2x}{(x^2-3)(\ln 2)}$$

4 good formulas to know:

$$\frac{d}{dx}\ln u = \frac{u'}{u}$$

$$\frac{d}{dx}e^u = e^u u'$$

$$\boxed{\frac{d}{dx}\log_a u = \frac{u'}{u\ln a}}$$

$$\boxed{\frac{d}{dx}a^u = a^u u'\ln a}$$

So going back to the problem on the previous screen, use the formula to find the derivative of $y = \log_2(x^2 - 3)$

$$y' = \frac{2x}{(x^2-3)\ln 2}$$

Use the formulas to differentiate the following.
(Hint: properties of logs still apply!!)

1) $y = 7^{(8x - 2)}$

$$y' = 7^{(8x-2)}(8)(\ln 7)$$

2) $y = \log_9 (8x^3\sqrt{x^2 - 10})$

$$y = \log_9 8x^3 + \frac{1}{2}\log_9 (x^2-10)$$

$$y' = \frac{24x^2}{8x^3 \ln 9} + \frac{1}{2} \cdot \frac{2x}{(x^2-10)\ln 9}$$

3) $y = \dfrac{e^{4x}}{2x + 6}$

$$y' = \frac{e^{4x}(4)(2x+6) - e^{4x}(2)}{(2x+6)^2}$$

Write the equation of the line tangent to y = ln (2x) at x = e.

pt: $(e, \ln(2e))$ or $(e, 1+\ln 2)$

$$y' = \frac{2}{2x} = \frac{1}{x}$$

$$y'(e) = \frac{1}{e}$$

$$y - \ln(2e) = \frac{1}{e}(x-e)$$ or

$$y - (1+\ln 2) = \frac{1}{e}(x-e)$$

Find dy/dx if ln(xy) = x + y

$$\ln x + \ln y = x + y$$

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} - \frac{dx}{dx} = 1 - \frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - 1\right) = 1 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1-\frac{1}{x}}{\frac{1}{y}-1} \cdot \frac{xy}{xy} = \frac{xy - y}{x - xy}$$

A particle moves along the x-axis so that at any time, t, its position is given by $x(t) = t^2e^{-t}$.

1) Find the velocity function, v(t). Then find v(1).

$$v(t) = 2te^{-t} + t^2e^{-t}(-1) = \frac{2t - t^2}{e^t}$$

$$v(1) = \frac{1}{e}$$

2) Find the acceleration function, a(t). Then find a(1).

$$a(t) = \frac{(2-2t)e^t - (2t-t^2)e^t}{e^{2t}} = \frac{2 - 4t + t^2}{e^t}$$

$$a(1) = -\frac{1}{e}$$

3) Find the time(s) when the particle is at rest and state why $v(t) = \frac{t(2-t)}{e^t}$ $t = 0, 2$ b/c $v(t) = 0$

4) State the interval(s) on which the particle is moving to the right and state why moving right on $(0,2)$ b/c $v(t) > 0$

$v(t)$: $-$ | $+$ | $-$

0 2

5) Is the velocity increasing or decreasing at t = 1? Why? velocity is decreasing at $t = 1$ b/c $a(1) < 0$

6) Is the speed increasing or decreasing at t = 1? Why? speed is decreasing at $t = 1$ b/c $a(1)$ and $v(1)$ have opposite signs

What have we learned?

- Can I switch back and forth between logarithmic and exponential forms?
- Can I differentiate a function involving a^x or $\log_a x$?

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