WARMUP!! (NO CALCULATORS!!)

1) Find dy/dx if

a) $y = \ln(7x)$

b) $y = \ln(x^3)$

c) $y = \ln(x - 3)$

d) $y = \ln(2x^3 + x)$

e) $y = \ln(\cos x)$

f) $y = \ln(\sin x)$

g) $y = \ln\sqrt{x^2 + 1}$

h) $y = \ln(1/(2x-1))$

2) Use logarithms to find dy/dx if

a) $y = \sqrt{x^2 + 1}$

b) $y = \dfrac{(x + 1)(x + 2)}{(x + 3)}$

1) a) $1/x$ b) $3/x$ c) $1/(x - 3)$

d) $(6x^2 + 1)/(2x^3 + x)$ e) $-\tan x$

f) $\cot x$ g) $x / (x^2 + 1)$ h) $-2/(2x - 1)$

2a) $(x\sqrt{x^2 + 1}) / (x^2 + 1) = x / \sqrt{x^2 + 1}$

2b) $\left(\dfrac{1}{x + 1} + \dfrac{1}{x + 2} - \dfrac{1}{x + 3}\right)\dfrac{(x + 1)(x + 2)}{x + 3}$

5.2b Integrals involving Natural Logs!

At the end of this lesson you will be able to:

> Use log integrals to review concepts such as definite integrals, area approximations and average value

It's all about u today! :)

17 $\displaystyle\int \frac{x^4+x-4}{x^2+2}\,dx$

$$\begin{array}{r|l} & x^2-2 \\ \hline x^2+0x+2 & x^4+0x^3+0x^2+x-4 \\ & -(x^4+0x^3+2x^2) \\ \hline & -2x^2+x-4 \\ & -(-2x^2+0x-4) \\ \hline & x \end{array}$$

$$= \int x^2-2+\frac{x}{x^2+2}\,dx$$

$u = x^2+2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{3}x^3-2x+\int \frac{1}{2}\cdot\frac{1}{u}\,du$$

$$= \frac{1}{3}x^3-2x+\frac{1}{2}\ln|x^2+2|+C$$

19 $\int \frac{(\ln x)^2}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$= \frac{1}{3}(\ln x)^3 + C$

25 $\int \frac{1}{1+\sqrt{2x}}\,dx = \int \frac{1}{u}\cdot\sqrt{2x}\,du$

$u = 1+\sqrt{2x}$

$du = \frac{1}{2}(2x)^{-\frac{1}{2}}(2)\,dx$

$du = \frac{1}{\sqrt{2x}}\,dx$

$\sqrt{2x}\,du = dx$

$\sqrt{2x} = u-1$

$= \int \frac{u-1}{u}\,du$

$= \int \left(1-\frac{1}{u}\right)du$

$= u - \ln|u| + C$

$= 1+\sqrt{2x} - \ln\left|1+\sqrt{2x}\right| + C$

$= \sqrt{2x} - \ln\left|1+\sqrt{2x}\right| + K$

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$$\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx = \int \frac{\sqrt{x}}{u}\cdot 2\sqrt{x}\,du$$

$u = \sqrt{x}-3$

$du = \frac{1}{2}x^{-\frac{1}{2}}dx$

$du = \frac{1}{2\sqrt{x}}dx$

$2\sqrt{x}\,du = dx$

$\rightarrow \sqrt{x} = u+3$

$x = (u+3)^2$

$$= \int \frac{2x}{u}\,du$$

$$= \int \frac{2(u+3)^2}{u}\,du$$

$$= \int \frac{2(u^2+6u+9)}{u}\,du$$

$$= \int \left(2u+12+\frac{18}{u}\right)du$$

$$= u^2+12u+18\ln|u|+C$$

$$= (\sqrt{x}-3)^2+12(\sqrt{x}-3)+18\ln|\sqrt{x}-3|+C$$

So you think you can integrate?

1) $\int \frac{1}{x\ln(x^3)}dx =$

4) $\int \frac{\sin^2 x - \cos^2 x}{\cos x}dx =$

2) $\int \frac{x}{x+3}dx =$

5) $\int 5x\csc(x^2+3)\,dx =$

3) $\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}dx =$

6) $\int \frac{x^3-6x-20}{x+5}dx =$

u

1) 1/3 ln| lnx | + C

2) x - 3ln|x + 3| + C

(note: the + 3 gets lumped with the C)

3) $(x^{1/3} - 1)^3 + 9/2(x^{1/3} - 1)^2 + 9(x^{1/3} - 1) + 3\ln|x^{1/3} - 1| + C$

4) ln|secx + tanx| - 2sinx + C

5) $-5/2 \ln|\csc(x^2 + 3) + \cot(x^2 + 3)| + C$

6) $(1/3)x^3 - (5/2)x^2 + 19x - 115\ln|x + 5| + C$

(3) $$\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\,dx = \int \frac{x^{\frac{1}{3}}}{u}\cdot 3x^{\frac{2}{3}}\,du$$

$u = \sqrt[3]{x} - 1$

$du = \frac{1}{3}x^{-\frac{2}{3}}\,dx$

$3x^{\frac{2}{3}}\,du = dx$

$\rightarrow \sqrt[3]{x} = u+1$

$x = (u+1)^3$

$$= \int \frac{3x}{u}\,du$$

$$= \int \frac{3(u+1)^3}{u}\,du$$

$$= \int \frac{3(u^3+3u^2+3u+1)}{u}\,du$$

$$= \int \left(3u^2+9u+9+\frac{3}{u}\right)du$$

$$= u^3 + \frac{9}{2}u^2 + 9u + 3\ln|u| + C$$

Keepin' it fresh!

Find the average value of the following:

1) $y = \dfrac{x^2+4}{x}$ on $[1,4]$

$$\text{Avg Val} = \frac{1}{4-1}\int_1^4 \frac{x^2+4}{x}\,dx = \frac{1}{3}\int_1^4 \left(x + \frac{4}{x}\right)dx$$

$$= \frac{1}{3}\left[\frac{1}{2}x^2 + 4\ln|x|\right]_1^4$$

$$= \frac{1}{3}\left[8 + 4\ln 4 - \left(\frac{1}{2}+0\right)\right]$$

$$= \frac{1}{3}\left[\frac{15}{2} + 4\ln 4\right]$$

2) $y = 2x - \tan x$ on $\left[0, \frac{\pi}{3}\right]$

$$\text{Avg} = \frac{1}{\frac{\pi}{3}-0}\int_0^{\frac{\pi}{3}} (2x - \tan x)\,dx$$

$$= \frac{3}{\pi}\left[x^2 + \ln|\cos x|\right]_0^{\frac{\pi}{3}}$$

$$= \frac{3}{\pi}\left[\frac{\pi^2}{9} + \ln\left(\frac{1}{2}\right) - \left(0 + \underbrace{\ln 1}_{0}\right)\right]$$

✓

1) $\frac{5}{2} + \frac{4}{3}\ln 4$

2) $\frac{\pi}{3} + \frac{3}{\pi}\ln\frac{1}{2}$

Suppose $f(x) = \dfrac{x}{x^2+4}$. Approximate the area between the curve and the x-axis from 0 to 4 using each of the following (make a table first). Then find the exact area. (Calculators permitted)

a) 4 left-handed rectangles

b) 4 right-handed rectangles

c) 2 midpoint rectangles

d) 4 trapezoids

e) Simpson's rule with n = 4

x	0	1	2	3	4
f(x)	0	1/5	1/4	3/13	1/5

a) 1[0 + 1/5 + 1/4 + 3/13] = 177/260 ≈ 0.681

b) 1[1/5 + 1/4 + 3/13 + 1/5] = 229/260 ≈ 0.881

c) 2[1/5 + 3/13] = 56/65 ≈ 0.862

d) (1/2)(1)[(0 + 1/5) + (1/5 + 1/4) + (1/4 + 3/13) + (3/13 + 1/5)] = 203/260 ≈ 0.781

e) (4/12)[0 + 4(1/5) + 2(1/4) + 4(3/13) + 1/5] = 21/26 ≈ 0.808

exact area =

$$\int_0^4 \frac{x}{x^2+4}\,dx = \frac{1}{2}\int_4^{20} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big|_4^{20} = \frac{1}{2}(\ln 20 - \ln 4) = \frac{1}{2}\ln 5 \approx 0.805$$

What have we learned?

- Can I remember how to solve problems involving average value, area approximation, really tough integrals and apply these concepts to lnx?